Parameter estimation for critical, symmetric 2-type Galton-Watson processes

Kristóf Körmendi and Gyula Pap (University of Szeged)

A Galton-Watson process with immigration is a stochastic process $(X_n)_{n\in\mathbb{N}}$ defined by the formula

$$X_n = \sum_{k=1}^{X_{n-1}} \xi_{n,k} + \varepsilon_n,$$

where X_n is the size of the n^{th} generation of a population, $\xi_{n,k}$ is the number of offsprings of the k^{th} individual belonging to the $(n-1)^{th}$ generation, and ε_n is the number of immigrants in the n^{th} generation.

We introduce the 2-type Galton-Watson process with immigration. We define the parameters in the form of a matirx m, where the element m_{ij} is the expectation of the number of j type offsprings of a single i type parent. We call the process symmetric if

$$m = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix},$$

and we call it critical if $\rho = \alpha + \beta = 1$ We propose conditional least square estimators for the paramters α, β and criticality parameter ρ, and prove the asymptotic properties of these estimators. We discuss the problems arising, when trying to generalize the results for the non-symmetric case.