Temporal cores in networks

Vladimir Batagelj
IMFM Ljubljana and IAM UP Koper

Monika Cerinšek
Abelium Ljubljana

XXXVIII Sunbelt 2018
Utrecht, The Netherlands, June 26 – July 1, 2018

Outline

Vladimir Batagelj: vladimir.batagelj@fmf.uni-lj.si

Current version of slides (June 30, 2018 at 04 : 44): Sunbelt'18 slides PDF

Cores

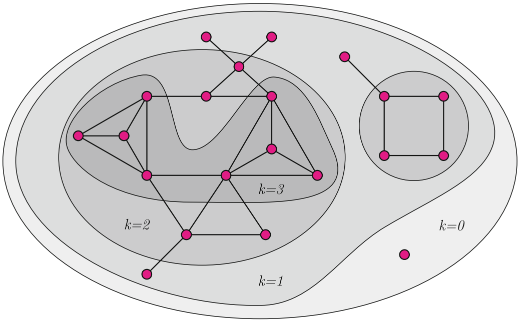
Network: $\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{P}, \mathcal{W})$;
$n = |\mathcal{V}|, m = |\mathcal{L}|$

A subgraph $\mathcal{H} = (\mathcal{C}, \mathcal{L}(\mathcal{C}))$ induced by the set of nodes $\mathcal{C}$ is a *k-core* or a core of order k iff $\forall v \in \mathcal{C} : deg_{\mathcal{H}}(v) \geq k$ and $\mathcal{H}$ is the maximum subgraph with this property (Seidman 1983).

The core of maximum order – main core.

The core number of node v is the highest order of a core that contains this node.

Batagelj and Zaveršnik (2003, 2011) proposed a very fast algorithm for determining core numbers.



Generalized cores

Node property function: $p(v, \mathcal{C}); v \in \mathcal{V}, \mathcal{C} \subseteq \mathcal{V}, p : L \longrightarrow \mathbb{R}^+$.

Properties:
$p(v, \mathcal{C})$ is *local*: $p(v, \mathcal{C}) = p(v, N(v, \mathcal{C})) \; \forall v \in \mathcal{V}$
$p(v, \mathcal{C})$ is *monotone*: $\mathcal{C}_1 \subset \mathcal{C}_2 \Rightarrow \forall v \in \mathcal{V} : p(v, \mathcal{C}_1) \leq p(v, \mathcal{C}_2)$.

The subgraph $\mathcal{H} = (\mathcal{C}, \mathcal{L}(\mathcal{C}))$ induced by the set $C \subseteq \mathcal{V}$ is a *p-core* at level $t \in \mathbb{R}$ iff $\forall v \in \mathcal{C} : t \leq p(v, \mathcal{C})$ and $\mathcal{C}$ is a maximal such set.

Generalized cores

Examples of node property function

1. $p_1(v, \mathcal{C}) = \deg_{\mathcal{C}}(v)$: node degree within $\mathcal{C}$
2. $p_2(v, \mathcal{C}) = \text{indeg}_{\mathcal{C}}(v) + \text{outdeg}_{\mathcal{C}}(v)$: if lines are directed it holds $p_2 = p_1$
3. $\mathbf{p_3(v, \mathcal{C}) = \sum_{u \in N(v, \mathcal{C})} w(v, u)}$ **for** $\mathbf{w : L \to \mathbb{R}_0^+}$ **: sum of weights of incident lines within** $\mathcal{C}$
4. $p_4(v, \mathcal{C}) = \max_{u \in N(v, \mathcal{C})} w(v, u)$ for $w : L \to \mathbb{R}$: maximal weight of incident lines within $\mathcal{C}$
5. $p_5(v, \mathcal{C}) = \frac{\deg_{\mathcal{C}}(v)}{\deg(v)}$ if $\deg(v) > 0$ else $f_5(v, \mathcal{C}) = 0$: fraction of neighbors within $\mathcal{C}$.
6. $p_6(v, \mathcal{C}) = \frac{\sum_{u \in N(v, \mathcal{C})} w(v,u)}{\sum_{u \in N(v)} w(v,u)}$ for $w : L \to \mathbb{R}_0^+$: fraction of sum of weights of incident lines within $\mathcal{C}$.

Temporal network

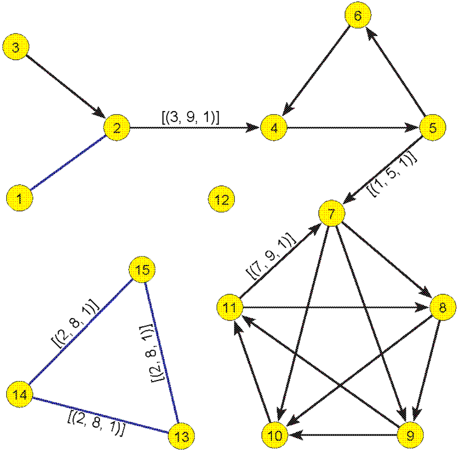
A *temporal network*

$$\mathcal{N}_\mathcal{T} = (\mathcal{V}, \mathcal{L}, \mathcal{T}, \mathcal{P}, \mathcal{W})$$

is obtained by attaching the time $\mathcal{T}$ to an ordinary network, where $\mathcal{T}$ is a set of time points: $t \in \mathcal{T}$ which are usually integers or reals.

Temporal quantities (TQ) are assigned to nodes and links:
a TQ is a list of triples (s, f, v) : s - start, f - finish of time interval $[s, f)$, v - value.

$T(v)$ – the activity set of time points for the node v; $T(l)$ the activity set of time points for the link l



Sum and product of temporal quantities

Temporal quantity a with the activity set $T_a \subseteq \mathcal{T}$ describes the changes of properties of nodes and links:

$$a = \begin{cases} a'(t) & t \in T_a \\ undefined & t \in \mathcal{T} \setminus T_a \end{cases}$$

Temporal quantities allow longitudinal approach instead of time slices.

```
a = [(1, 5, 2), (6, 8, 1), (11, 12, 3), (14, 16, 2), (17, 18, 5), (19, 20, 1)]
b = [(2, 3, 4), (4, 7, 3), (9, 10, 2), (13, 15, 5), (16, 21, 1)]
```

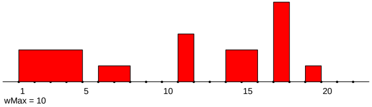
The following are the sum $s = a + b$ and the product $p = a \cdot b$ of temporal quantities a and b over combinatorial semiring.

```
s = [(1, 2, 2), (2, 3, 6), (3, 4, 2), (4, 5, 5), (5, 6, 3), (6, 7, 4), (7, 8, 1),
     (9, 10, 2), (11, 12, 3), (13, 14, 5), (14, 15, 7), (15, 16, 2), (16, 17, 1),
     (17, 18, 6), (18, 19, 1), (19, 20, 2), (20, 21, 1)]
p = [(2, 3, 8), (4, 5, 6), (6, 7, 3), (14, 15, 10), (17, 18, 5), (19, 20, 1)]
```

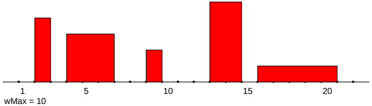
They are visually displayed at the bottom half of figures on the following slides.

Addition and multiplication of temporal quantities

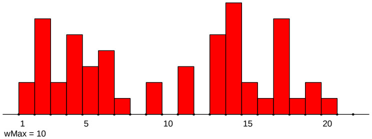
a :



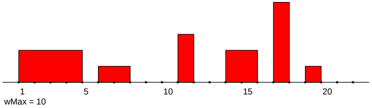
b :



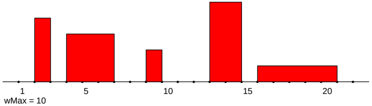
$a + b$:



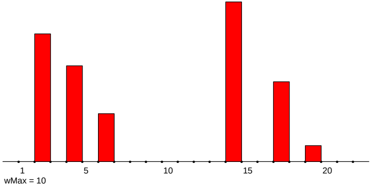
a :



b :



$a \cdot b$:



Core decomposition

```
CoreDecomposition(𝒩):
C = V
k = 1
while C ≠ ∅:
        while ∃ u ∈ C ∋: deg(u) < k:
                for v ∈ N(u, C):
                        deg(v) = deg(v) − 1
                C = C\v
                core(u) = k − 1
        k = k + 1
```

We extended this algorithm to ordinary temporal cores and temporal p_S-cores.
For programs in Python see GitHub/Bavla/Graph.

Algorithm for ordinary temporal cores

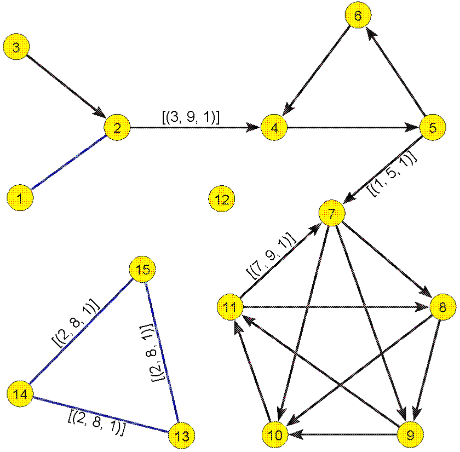
```
TemporalCores(𝒩):
D = {u: [triples (start, finish, deg)]}
CoreHierarchy = {u: [triples with deg = 0]}
D = (D.filter(deg > 0)).remove(empty triples)
Dmin = {u: min deg}
while D not empty:
        (dmin, u) = (deg, u) ∋: (u, deg) ∈ Dmin ∧ deg is min deg
        core = [triples from D[u] ∋: deg[u] from triple is equal to dmin]
        CoreHierarchy[u].add(core)
        change = core.set(deg = −1)
        D[u] = D[u].add(change).cutAt(dmin) \\ value >= dmin
        for l in 𝒩.star(u):
                v = other end−node of l
                if not v in D: continue
                changeLink = l.intersection(change).set(deg = −1)
                if changeLink empty: continue
                diff = D[v].add(changeLink).cutAt(0) \\ value >= 0
                D[v] = diff.set(max(currentValue, dmin))
                if D[v] is empty:
                        delete D[v], Dmin[v]
                else:
                        Dmin[v] = triple ∈ D[v] with min deg
        if D[u] empty:
                delete D[u], Dmin[u]
        else:
                Dmin[u] = triple ∈ D[u] with min deg
return CoreHierarchy
```

Algorithm for p_S temporal cores

```
PSTemporalCores(𝒩):
D = {u: [triples (start, finish, weightSum)]}
Core = {u: [triples with weightSum = 0]}
D = (D.filter(weightSum > 0)).remove(empty triples)
Dmin = {u: min weightSum}
while D not empty:
    (dmin, u) = (weightSum, u) ∋: Dmin ∧ weightSum is min weightSum
    core = [triples from D[u] ∋: weightSum[u] from triple is equal to dmin]
    if core not empty:
        Core[u].add(core)
        change = core.set(weightSum = −weightSum)
        D[u] = D[u].add(change).cutAt(dmin) \\ value >= dmin
        for l in 𝒩.star(u):
            v = other end−node of l
            if not v in D: continue
            changeLink = l.intersection(change).set(weightSum = −weightSum)
            if changeLink empty: continue
            diff = D[v].add(changeLink).cutAt(0) \\ value >= 0
            D[v] = diff.set(max(currentValue, dmin))
            if D[v] is empty:
                delete D[v], Dmin[v]
            else:
                Dmin[v] = triple ∈ D[v] with min weightSum
    if D[u] is empty:
        delete D[u], Dmin[u]
    else:
        Dmin[u] = triple ∈ D[u] with min weightSum
return Core
```

Artificial example

all weights $w = 1$



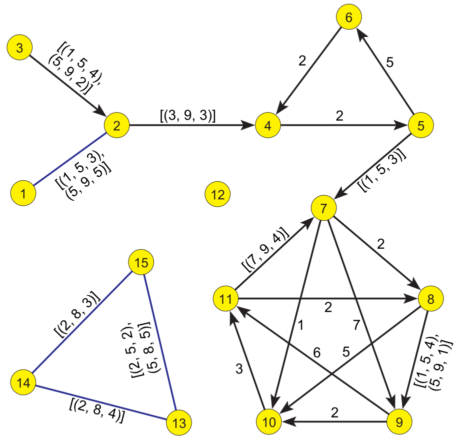
Artificial example

all weights $w = 1$

Node	Degree	Core number
1	(1, 9, 1)	(1, 9, 1)
2	(1, 3, 2), (3, 9, 3)	(1, 9, 1)
3	(1, 9, 1)	(3, 9, 1)
4	(1, 3, 2), (3, 9, 3)	(1, 9, 2)
5	(1, 5, 3), (5, 9, 2)	(1, 9, 2)
6	(1, 9, 2)	(1, 9, 2)
7	(1, 5, 4), (5, 7, 3), (7, 9, 4)	(1, 7, 3), (7, 9, 4)
8	(1, 9, 4)	(1, 7, 3), (7, 9, 4)
9	(1, 9, 4)	(1, 7, 3), (7, 9, 4)
10	(1, 9, 4)	(1, 7, 3), (7, 9, 4)
11	(1, 7, 3), (7, 9, 4)	(1, 7, 3), (7, 9, 4)
12	(1, 9, 0)	(1, 9, 0)
13	(1, 2, 0), (2, 8, 2), (8, 9, 0)	(1, 2, 0), (2, 8, 2), (8, 9, 0)
14	(1, 2, 0), (2, 8, 2), (8, 9, 0)	(1, 2, 0), (2, 8, 2), (8, 9, 0)
15	(1, 2, 0), (2, 8, 2), (8, 9, 0)	(1, 2, 0), (2, 8, 2), (8, 9, 0)

Artificial example

different weights



Artificial example

different weights

Node	Degree	Core number
1	(1, 5, 3), (5, 9, 5)	(1, 5, 3), (5, 9, 5)
2	(1, 3, 7), (3, 9, 10)	(1, 5, 4), (5, 9, 5)
3	(1, 5, 4), (5, 9, 2)	(1, 5, 4), (5, 9, 2)
4	(1, 3, 4), (3, 9, 7)	(1, 5, 4), (5, 9, 5)
5	(1, 5, 10), (5, 9, 7)	(1, 9, 5)
6	(1, 9, 7)	(1, 9, 5)
7	(1, 5, 13), (5, 7, 10), (7, 9, 14)	(1, 9, 10)
8	(1, 5, 13), (5, 9, 10)	(1, 9, 10)
9	(1, 5, 19), (5, 9, 16)	(1, 9, 10)
10	(1, 9, 11)	(1, 9, 10)
11	(1, 7, 11), (7, 9, 15)	(1, 9, 10)
12	(1, 9, 0)	(1, 9, 0)
13	(1, 2, 0), (2, 5, 6), (5, 8, 9), (8, 9, 0)	(1, 2, 0), (2, 5, 5), (5, 8, 7), (8, 9, 0)
14	(1, 2, 0), (2, 8, 7), (8, 9, 0)	(1, 2, 0), (2, 5, 5), (5, 8, 7), (8, 9, 0)
15	(1, 2, 0), (2, 5, 5), (5, 8, 8), (8, 9, 0)	(1, 2, 0), (2, 5, 5), (5, 8, 7), (8, 9, 0)

Reuters terror news network

Obtained from the CRA (Centering Resonance Analysis) networks produced by Steve Corman and Kevin Dooley at Arizona State University.

Based on all the stories released during 66 consecutive days by the news agency Reuters concerning the September 11 attack on the U.S., beginning at 9:00 AM EST 9/11/01.

Nodes: important words (terms), n =13332

Links: two nodes appear in the same utterance, $m = 243447$, undirected, weight is equal to the frequency of appearance, 50859 of them have the weight larger than 1. No loops.

Data available at: Terror.

Example: induced subnetwork on 50 most active nodes.

Reuters terror news network

Temporal degrees

Node	**Degree**
1	(1, 2, 5), (2, 3, 6), (3, 4, 3), (4, 5, 5), (5, 6, 4), (6, 8, 3), (8, 10, 5), (10, 11, 3), (11, 13, 2), (13, 16, 3), (16, 17, 4), (17, 18, 5), (18, 19, 3), (19, 21, 1), (21, 22, 2), (22, 23, 1), (23, 24, 4), (24, 25, 1), (25, 29, 3), (29, 31, 2), (31, 33, 3), (33, 34, 1), (34, 36, 3), (36, 37, 2), (37, 39, 3), (39, 40, 4), (40, 41, 2), (41, 42, 0), (42, 43, 3), (43, 44, 2), (44, 45, 3), (45, 46, 1), (46, 47, 2), (47, 48, 3), (48, 49, 0), (49, 50, 4), (50, 51, 1), (51, 52, 2), (52, 53, 1), (53, 54, 0), (54, 58, 2), (58, 59, 3), (59, 60, 2), (60, 61, 4), (61, 62, 0), (62, 64, 2), (64, 65, 1), (65, 67, 2)
2	(1, 2, 27), (2, 3, 29), ..., (63, 64, 2), (64, 65, 0), (66, 67, 0)
...	
50	(1, 2, 3), (2, 3, 2), (3, 5, 1), (5, 8, 0), (8, 10, 1), (10, 11, 2), (11, 12, 1), (12, 15, 0), (15, 16, 3), (16, 17, 1), (17, 19, 0), (19, 20, 1), (20, 21, 2), (21, 22, 0), (22, 24, 1), (24, 26, 0), (26, 27, 2), (27, 28, 0), (28, 29, 1), (29, 31, 0), (31, 32, 1), (32, 33, 0), (33, 35, 1), (35, 37, 0), (37, 38, 1), (38, 42, 0), (43, 44, 2), (44, 49, 0), (49, 50, 2), (51, 57, 0), (58, 61, 0), (61, 62, 1), (62, 67, 0)

Node	Core number
1	(1, 2, 4), (2, 3, 5), (3, 5, 3), (5, 6, 4), (6, 8, 3), (8, 10, 4), (10, 11, 3), (11, 14, 2), (14, 18, 3), (18, 19, 2), (19, 21, 1), (21, 22, 2), (22, 23, 1), (23, 24, 3), (24, 25, 1), (25, 28, 2), (28, 29, 3), (29, 33, 2), (33, 34, 1), (34, 38, 2), (38, 39, 3), (39, 41, 2), (41, 42, 0), (42, 45, 2), (45, 46, 1), (46, 47, 2), (47, 48, 3), (48, 49, 0), (49, 50, 3), (50, 51, 1), (51, 52, 2), (52, 53, 1), (53, 54, 0), (54, 57, 2), (57, 58, 1), (58, 59, 2), (59, 60, 1), (60, 61, 2), (61, 62, 0), (62, 64, 2), (64, 65, 1), (65, 67, 2)
2	(1, 3, 5), (3, 6, 4), (6, 7, 5), ..., (63, 64, 1), (64, 65, 0), (66, 67, 0)
...	
50	(1, 3, 2), (3, 5, 1), (5, 8, 0), (8, 10, 1), (10, 11, 2), (11, 12, 1), (12, 15, 0), (15, 16, 3), (16, 17, 1), (17, 19, 0), (19, 20, 1), (20, 21, 2), (21, 22, 0), (22, 24, 1), (24, 26, 0), (26, 27, 1), (27, 28, 0), (28, 29, 1), (29, 31, 0), (31, 32, 1), (32, 33, 0), (33, 35, 1), (35, 37, 0), (37, 38, 1), (38, 42, 0), (43, 44, 1), (44, 49, 0), (49, 50, 2), (51, 57, 0), (58, 61, 0), (61, 62, 1), (62, 67, 0)

Reuters terror news network

Temporal cores of order at least 3 appear in the first 11 days and on 30th day

	Node	Core number (≥ 3)
25	world	(1, 3, 5), (3, 10, 4)
2	attack	(1, 3, 5), (3, 6, 4), (6, 7, 5), (7, 10, 4), (11, 12, 4), (30, 31, 4)
9	washington	(1, 3, 5), (3, 6, 4), (6, 7, 5), (7, 10, 4), (11, 12, 4)
14	world_trade_ctr	(1, 3, 5), (3, 6, 4), (6, 7, 5), (30, 31, 4)
4	people	(1, 3, 5), (3, 6, 4), (6, 7, 5), (7, 8, 4)
21	pentagon	(1, 3, 5), (3, 4, 4), (5, 6, 4), (6, 7, 5)
7	new_york	(1, 3, 5), (3, 6, 4), (6, 7, 5), (30, 31, 4)
8	pres_bush	(1, 3, 5), (3, 6, 4), (6, 7, 5), (7, 10, 4), (11, 12, 4)
10	official	(1, 3, 5), (3, 4, 4), (5, 6, 4), (6, 7, 5)
43	tower	(1, 3, 5), (3, 4, 4), (6, 7, 5)
34	time	(1, 3, 5), (3, 4, 4), (5, 6, 4), (7, 8, 4)
18	city	(1, 3, 5), (3, 4, 4)
20	tuesday	(1, 3, 5), (3, 7, 4)
13	plane	(1, 3, 5), (3, 7, 4)
15	security	(1, 2, 4), (2, 3, 5), (5, 6, 4)
1	united_states	(1, 2, 4), (2, 3, 5), (5, 6, 4), (8, 10, 4)
19	war	(1, 2, 4), (2, 3, 5), (5, 8, 4)
29	worker	(1, 2, 4), (2, 3, 5)
47	wednesday	(2, 3, 5), (3, 4, 4), (8, 10, 4)
12	military	(1, 2, 4), (5, 6, 4), (30, 31, 4)
5	afghanistan	(1, 3, 4), (5, 6, 4), (6, 7, 5), (8, 10, 4), (30, 31, 4)

Reuters terror news network

Temporal cores of order at least 3 appear in the first 11 days and on 30th day

	Node	Core number (≥ 3)
6	bin_laden	(1, 4, 4), (5, 6, 4), (6, 7, 5), (7, 10, 4), (11, 12, 4)
36	strike	(2, 3, 4), (5, 6, 4), (6, 7, 5), (30, 31, 4)
28	week	(5, 6, 4), (6, 7, 5), (8, 10, 4), (11, 12, 4)
48	nation	(1, 3, 4), (5, 6, 4)
40	terrorist	(1, 3, 4), (6, 7, 4)
17	country	(1, 3, 4), (5, 10, 4)
23	government	(1, 3, 4), (5, 6, 4)
30	office	(1, 3, 4)
24	leader	(1, 4, 4), (6, 10, 4)
49	police	(2, 4, 4), (5, 6, 4)
31	group	(2, 3, 4), (6, 7, 4)
42	pakistan	(2, 3, 4), (5, 7, 4)
32	air	(2, 3, 4), (5, 6, 4)
27	day	(2, 3, 4), (5, 6, 4)
35	hijack	(2, 3, 4)
26	terrorism	(2, 3, 4)
38	flight	(2, 3, 4)
39	tell	(2, 3, 4)
16	american	(2, 3, 4)
41	airport	(2, 3, 4)
45	new	(2, 3, 4)
22	force	(5, 6, 4)

Reuters terror news network

Temporal p_S-cores

	Node	p_S-core number (≥ 20)
2	attack	(1, 3, 86), (3, 4, 44), (4, 5, 36), (5, 6, 66), (6, 7, 47), (7, 8, 22), (8, 9, 21), (9, 10, 24), (10, 11, 22), (11, 12, 24), (15, 16, 23), (18, 19, 20), (27, 28, 23)
7	new_york	(1, 2, 101), (2, 3, 86), (3, 4, 42), (4, 5, 35), (5, 6, 66), (6, 7, 47), (8, 9, 21), (9, 10, 24), (10, 11, 22), (11, 12, 24), (15, 16, 23), (18, 19, 20)
14	world_trade_c	(1, 2, 101), (2, 3, 86), (3, 4, 44), (4, 5, 35), (5, 6, 66), (6, 7, 47), (8, 9, 20), (9, 10, 21), (10, 11, 22), (11, 12, 24), (15, 16, 23), (18, 19, 20)
9	washington	(1, 2, 80), (2, 3, 61), (3, 4, 27), (4, 5, 28), (5, 6, 66), (6, 7, 47), (8, 9, 21), (9, 10, 24), (10, 11, 22), (11, 12, 24), (15, 16, 23), (18, 19, 20)
21	pentagon	(1, 3, 86), (3, 4, 44), (4, 5, 32), (5, 6, 66), (6, 7, 47), (8, 9, 20), (9, 10, 21), (10, 11, 22), (11, 12, 24), (15, 16, 23), (18, 19, 20)
1	united_states	(1, 2, 86), (2, 3, 71), (3, 4, 34), (4, 5, 29), (5, 6, 50), (6, 7, 47), (7, 8, 22), (15, 16, 23), (18, 19, 23), (27, 28, 23)
28	week	(5, 6, 35), (6, 7, 27), (7, 8, 22), (8, 9, 21), (9, 10, 24), (10, 11, 22), (11, 12, 24)
4	people	(1, 2, 48), (2, 3, 52), (3, 4, 28), (4, 5, 32), (5, 6, 29), (6, 7, 34), (18, 19, 20)
12	military	(1, 2, 25), (2, 3, 42), (5, 6, 26), (15, 16, 23), (18, 19, 23), (27, 28, 23)
5	afghanistan	(1, 2, 22), (2, 3, 28), (5, 6, 29), (6, 7, 21), (15, 16, 23), (27, 28, 23)
6	bin_laden	(1, 2, 22), (2, 3, 28), (3, 4, 20), (5, 6, 29), (6, 7, 21), (18, 19, 20)
10	official	(1, 2, 40), (2, 3, 54), (3, 4, 34), (5, 6, 29), (6, 7, 36), (18, 19, 23)
43	tower	(1, 2, 101), (2, 3, 72), (3, 4, 41), (4, 5, 32), (5, 6, 38), (6, 7, 32)
35	hijack	(1, 2, 67), (2, 3, 86), (3, 4, 44), (4, 5, 28), (5, 6, 50), (6, 7, 34)
13	plane	(1, 3, 86), (3, 4, 44), (4, 5, 32), (5, 6, 50), (6, 7, 34)

Reuters terror news network

Temporal p_S-cores

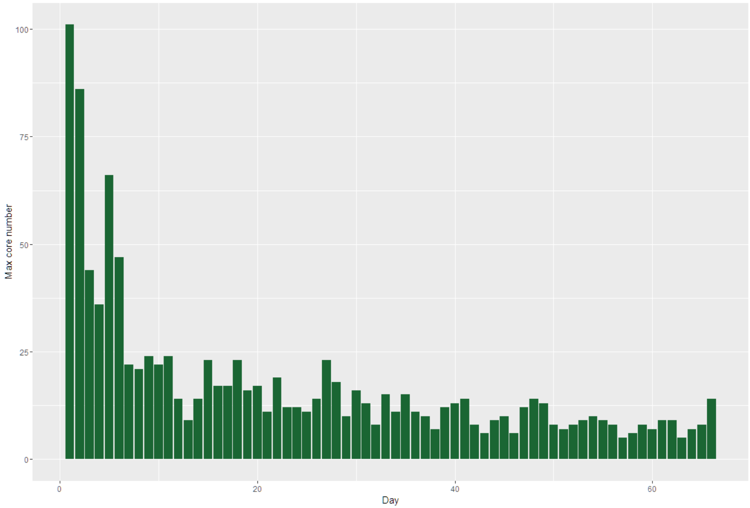
	Node	p_S-core number (≥ 20)
20	tuesday	(1, 3, 86), (3, 4, 44), (4, 5, 36), (5, 6, 66), (6, 7, 47)
3	taliban	(2, 3, 28), (6, 7, 20), (15, 16, 23), (27, 28, 23)
36	strike	(2, 3, 29), (5, 6, 29), (18, 19, 22), (27, 28, 23)
17	country	(1, 2, 24), (2, 3, 31), (5, 6, 26), (18, 19, 20)
8	pres_bush	(1, 2, 48), (2, 3, 44), (5, 6, 29), (6, 7, 21)
41	airport	(1, 2, 25), (2, 3, 44), (4, 5, 25), (5, 6, 24)
15	security	(1, 2, 25), (2, 3, 30), (5, 6, 24)
16	american	(1, 2, 48), (2, 3, 30), (5, 7, 20)
18	city	(1, 2, 60), (2, 3, 52), (3, 4, 22)
25	world	(1, 2, 34), (2, 3, 44), (18, 19, 20)
27	day	(1, 2, 21), (2, 3, 36), (5, 6, 20)
32	air	(2, 3, 34), (5, 6, 29), (27, 28, 23)
38	flight	(1, 2, 25), (2, 3, 52), (4, 5, 20)
48	nation	(1, 2, 31), (2, 3, 38), (5, 6, 23)
40	terrorist	(1, 2, 40), (2, 3, 29)
19	war	(2, 3, 34), (5, 6, 29)
23	government	(1, 2, 28), (2, 3, 36)
46	buildng	(1, 2, 34), (2, 3, 44)
30	office	(1, 2, 34), (2, 3, 20)

Reuters terror news network

Temporal p_S-cores

	Node	p_S-core number ($\geq$ 20)
26	terrorism	(5, 6, 20)
29	worker	(1, 2, 24)
31	group	(2, 3, 26)
34	time	(2, 3, 36)
22	force	(5, 6, 26)
24	leader	(1, 2, 22)
42	pakistan	(5, 6, 29)
44	bomb	(1, 2, 23)
45	new	(2, 3, 30)
47	wednesday	(2, 3, 52)
49	police	(2, 3, 20)

Max p_S-core numbers by days from the event



Franzosi's Violence network

Roberto Franzosi collected from the journal news in the period January 1919 - December 1922 information about the different types of interactions between political parties and other groups of people in Italy. The violence network contains only the data about violent actions and counts the number of interactions per month.

Nodes: groups of people, $n = 29$

Links: violent interactions, $m = 105$

For details see:
Franzosi, R., 1997. Mobilization and CounterMobilization Processes: From the Red Years (1919-20) to the Black Years (1921-22) in Italy.
Franzosi, R., 1997. A New Methodological Approach to the Study of Narrative Data. Theory and Society, 26(2-3), 275-304

Violence network

Core number ≥ 3

Node		**Core number** (≥ 3)
16	workers	(29, 30, 3), (33, 34, 3), (39, 41, 3)
1	undefined	(29, 30, 3), (39, 40, 3)
2	?	(31, 32, 3), (33, 34, 3), (40, 41, 3)
3	people	(31, 32, 3), (33, 34, 3), (39, 40, 3)
4	police	(31, 32, 3), (33, 34, 3), (40, 41, 3)
21	catholics	(33, 34, 3)
7	fascists	(29, 30, 3), (31, 32, 3), (33, 34, 3), (39, 41, 3)
8	communists	(29, 30, 3)
10	socialists	(31, 32, 3), (40, 41, 3)

Violence network

Core number ≥ 2

Node		**Core number** (≥ 2)
1	undefined	(15, 16, 2), (17, 18, 2), (25, 29, 2), (29, 30, 3), (31, 32, 2), (38, 39, 2), (39, 40, 3), (41, 44, 2), (45, 46, 2), (48, 49, 2)
2	?	(14, 16, 2), (17, 18, 2), (28, 29, 2), (31, 32, 3), (32, 33, 2), (33, 34, 3), (34, 35, 2), (40, 41, 3)
3	people	(16, 18, 2), (23, 24, 2), (25, 26, 2), (28, 30, 2), (31, 32, 3), (33, 34, 3), (35, 37, 2), (39, 40, 3), (41, 43, 2), (48, 49, 2)
4	police	(11, 12, 2), (14, 20, 2), (21, 23, 2), (29, 31, 2), (31, 32, 3), (32, 33, 2), (33, 34, 3), (34, 37, 2), (38, 40, 2), (40, 41, 3)
5	land owners	(15, 16, 2), (17, 20, 2), (29, 30, 2), (36, 37, 2), (38, 40, 2), (42, 43, 2)
7	fascists	(11, 12, 2), (16, 17, 2), (19, 20, 2), (21, 24, 2), (25, 29, 2), (29, 30, 3), (30, 31, 2), (31, 32, 3), (32, 33, 2), (33, 34, 3), (34, 37, 2), (38, 39, 2), (39, 41, 3), (41, 44, 2), (45, 46, 2), (48, 49, 2)
8	communists	(28, 29, 2), (29, 30, 3), (31, 33, 2), (35, 37, 2), (43, 44, 2)
9	workers (agr)	(15, 16, 2), (17, 20, 2), (28, 30, 2), (31, 32, 2), (33, 35, 2), (38, 43, 2), (45, 46, 2)
10	socialists	(11, 12, 2), (16, 18, 2), (19, 20, 2), (22, 23, 2), (25, 26, 2), (27, 30, 2), (31, 32, 3), (33, 37, 2), (38, 40, 2), (40, 41, 3), (41, 42, 2)
12	war affected	(35, 36, 2), (39, 40, 2)
13	protesters	(15, 16, 2), (21, 22, 2), (29, 30, 2), (31, 32, 2), (38, 40, 2)

Violence network

Core number ≥ 2

Node		**Core number** (≥ 2)
16	workers	(11, 12, 2), (14, 18, 2), (19, 20, 2), (21, 24, 2), (25, 26, 2), (27, 29, 2), (29, 30, 3), (30, 33, 2), (33, 34, 3), (34, 37, 2), (38, 39, 2), (39, 41, 3), (41, 44, 2), (45, 46, 2)
17	the right	(17, 18, 2), (41, 42, 2)
19	populars	(41, 42, 2)
20	students	(17, 18, 2)
21	catholics	(33, 34, 3)
25	republicans	(26, 27, 2)
26	thugs	(29, 30, 2)
27	prisoners/arrested	(40, 41, 2)

Violence network

Node		p_S-ore number (≥ 10)
16	workers	(1, 2, 27), (10, 11, 11), (14, 15, 27), (16, 17, 11), (17, 18, 17), (18, 19, 12), (22, 23, 17), (25, 26, 11), (27, 28, 18), (28, 29, 16), (29, 30, 53), (30, 31, 56), (31, 32, 51), (32, 33, 30), (33, 34, 17), (34, 35, 71), (35, 36, 76), (36, 37, 53), (37, 38, 11), (38, 39, 23), (39, 40, 54), (40, 41, 13), (41, 42, 174), (42, 43, 25), (43, 44, 20), (45, 46, 15), (46, 47, 25)
10	socialists	(10, 11, 10), (12, 13, 29), (27, 28, 30), (28, 29, 31), (29, 30, 64), (30, 31, 29), (31, 32, 17), (32, 33, 14), (33, 34, 24), (34, 35, 38), (35, 36, 23), (36, 37, 26), (37, 38, 13), (38, 39, 19), (39, 40, 54), (45, 46, 13)
4	police	(1, 2, 36), (6, 7, 15), (10, 11, 24), (12, 13, 29), (14, 15, 27), (15, 16, 13), (16, 17, 24), (17, 18, 17), (18, 19, 12), (22, 23, 17), (31, 32, 17)
7	fascists	(25, 26, 11), (27, 28, 30), (28, 29, 31), (29, 30, 64), (30, 31, 56), (31, 32, 51), (32, 33, 30), (33, 34, 24), (34, 35, 71), (35, 36, 76), (36, 37, 53), (37, 38, 13), (38, 39, 23), (39, 40, 54), (40, 41, 13), (41, 42, 174), (42, 43, 25), (43, 44, 20), (45, 46, 15), (46, 47, 25)
9	workers (agr)	(10, 11, 24), (16, 17, 24), (28, 29, 16), (30, 31, 13), (36, 37, 11), (39, 40, 15), (43, 44, 10)
1	undefined	(25, 26, 11), (27, 28, 12), (28, 29, 16), (41, 42, 133), (45, 46, 11)
8	communists	(29, 30, 13), (30, 31, 10), (31, 32, 12)
13	protesters	(6, 7, 15), (15, 16, 13), (16, 17, 20)
12	war affected	(1, 2, 36)
3	people	(28, 29, 12)

SN5 network

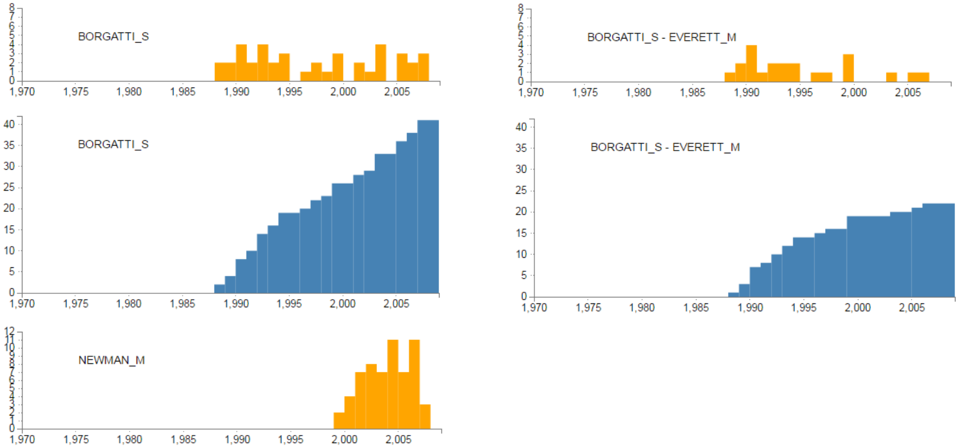
Data from Web of Science (`"social network*" AND SO=(Social networks)`) plus most frequently cited works plus around 100 SNA researchers. Collected in December 2007 for the 2008 Viszards session.

We analyze the works $\times$ authors network **WA** restricted to works with a complete description ($DC > 0$): $|\mathbf{W}| = 7950$, $|\mathbf{A}| = 12458$ and $m = 19488$.

Using the publication years the network **WA** was expanded to a temporal network with cumulative weights.

Normalization "by rows": $\mathbf{N} = n(\mathbf{WA})$.
Normalized coauthorship network: $\mathbf{Ct} = \mathbf{N}^T * \mathbf{N}$.

SN5 Coauthorship



SN5 cumulative coauthorship p_S cores ≥ 3


```
  20 : BORGATTI_S   [(1991, 1992, 3.1667), (1992, 1993, 4.1667), (1993, 1994, 5.1667),
    (1994, 1996, 6.1667), (1996, 1997, 6.6667), (1997, 1999, 7.1667), (1999, 2003, 8.6667),
    (2003, 2005, 8.7917), (2005, 2006, 9.2917), (2006, 2009, 9.7917)]
3169 : EVERETT_M    [(1991, 1992, 3.1667), (1992, 1993, 4.1667), (1993, 1994, 5.1667),
    (1994, 1996, 6.1667), (1996, 1997, 6.6667), (1999, 2003, 8.6667), (2003, 2005, 8.7917),
    (2005, 2006, 9.2917), (2006, 2009, 9.7917)]
 317 : BERNARD_H    [(1990, 1991, 3.0244), (1991, 1995, 3.1494), (1995, 1997, 3.3094),
    (1997, 1998, 3.3894), (1998, 2001, 3.5494), (2001, 2003, 3.6294), (2003, 2006, 3.685),
    (2006, 2009, 4.0706)]
2232 : KILLWORT_P   [(1990, 1991, 3.0244), (1991, 1995, 3.1494), (1995, 1997, 3.3094),
    (2003, 2006, 3.685), (2006, 2009, 4.0706)]
4551 : STEINHAU_H   [(2003, 2005, 3.0), (2005, 2006, 3.2222), (2006, 2009, 3.6667)]
4860 : METZKE_C     [(2003, 2005, 3.0), (2005, 2006, 3.2222), (2006, 2009, 3.6667)]
3125 : SHELLEY_G    [(2006, 2009, 3.4767)]
1673 : MCCARTY_C    [(2006, 2009, 3.4767)]
1677 : JOHNSEN_E    [(2006, 2009, 3.4767)]
  75 : HOLLAND_P    [(1981, 1983, 3.0), (1983, 2009, 3.2222)]
  78 : LEINHARD_S   [(1981, 1983, 3.0), (1983, 2009, 3.2222)]
 925 : BONACICH_P   [(1997, 2009, 3.2222)]
3840 : BIENENST_E   [(1997, 2009, 3.2222)]
  69 : WASSERMA_S   [(2007, 2009, 3.0174)]
1164 : DOREIAN_P    [(2007, 2009, 3.0174)]
1166 : HUMMON_N     [(2007, 2009, 3.0174)]
1680 : PATTISON_P   [(2007, 2009, 3.0174)]
3225 : FARARO_T     [(2007, 2009, 3.0174)]
1056 : FAUST_K      [(2007, 2009, 3.0174)]
3170 : FERLIGOJ_A   [(2007, 2009, 3.0174)]
2083 : ROBINS_G     [(2007, 2009, 3.0174)]
2084 : SKVORETZ_J   [(2007, 2009, 3.0174)]
 949 : BATAGELJ_V   [(2007, 2009, 3.0174)]
  79 : NEWMAN_M     [(2005, 2009, 3.0)]
 796 : PARK_J       [(2005, 2009, 3.0)]
```

Conclusions

1. Improve the complexity of the algorithm
2. Extend the algorithm to generalized temporal cores
3. Find user friendly presentations of results
4. Compare with the streaming core algorithms

Temporal Quantities – a Python 3 library for temporal network analysis TQ / Graph.

Acknowledgements.
The first author was partially sponsored by Slovenian Research Agency (ARRS) – projects J7-8279 and J1-6720, and grant P1-0294.
The second author was partially sponsored by Slovenian Research Agency (ARRS) – projects Z7-7614 (B) and L7-5554.

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