Cores in temporal networks / Longitudinal approach to core maintenance

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Abstract

TODO: zaenkrat enako kot na konferenci

Among many formalizations of the notion of dense subnetworks of a given network (clique, s-plexes, LS sets, lambda sets, cores, etc.) only for cores exists an efficient algorithm that can compute the result in an acceptable time also for large networks.

In a graph $G = (V, E)$ a subgraph $H = (W, E(W))$ induced by the subset of nodes W is a k-core or a core of order k iff for each node v in W its internal degree $\deg_H(v)$ is greater or equal to k, and H is a maximal subgraph with this property.

Various generalizations of ordinary cores according to types of networks and measures of node importance were proposed. In our contribution we extend the notion of cores to temporal networks and present the corresponding algorithms. For illustration we present results of determining cores in selected artificial and real-life networks.

Keywords:
Mathematics Subject Classification:

1 Introduction

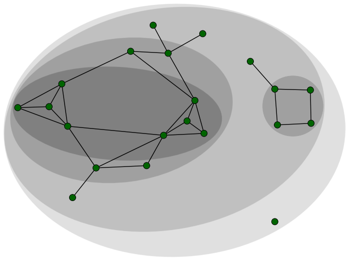
Core of order k

Network: $\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{P}, \mathcal{W})$; $n = |\mathcal{V}|, m = |\mathcal{L}|$

k-core (Seidman 1983): A subgraph $\mathcal{H} = (\mathcal{C}, \mathcal{L}(\mathcal{C}))$ induced by the set of nodes $\mathcal{C}$ is a k-core or a core of order k iff $\forall v \in \mathcal{C} : deg_{\mathcal{H}}(v) \geq k$ and $\mathcal{H}$ is the maximum subgraph with this property.

The core of maximum order – main core.

The core number of node v is the highest order of a core that contains this node.



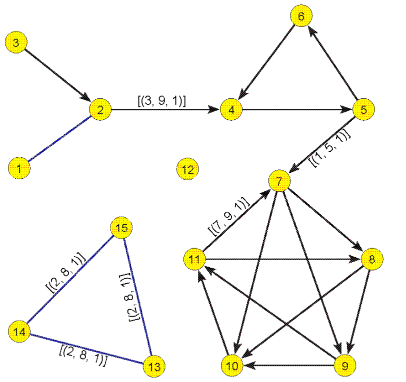
1.1 Temporal network and temporal cores

Temporal network

Temporal network $\mathcal{N}_{\mathcal{T}} = (\mathcal{V}, \mathcal{L}, \mathcal{T}, \mathcal{P}, \mathcal{W})$ is obtained by attaching the time $\mathcal{T}$ to an ordinary network, where $\mathcal{T}$ is a set of time points: $t \in \mathcal{T}$ which are usually integers or reals.

Temporal quantities assigned to nodes and links:

a TQ is a list of triples (s,f,v) : s - start, f - finish of time interval [s,f), v - value



Temporal quantities Notion: $T(v)$ – the activity set of time points for the node v; $T(l)$ the activity set of time points for the link l

Consistency condition: If a link $l(u, v)$ is active at the time point t then its end-nodes u and v should be active at the time t :

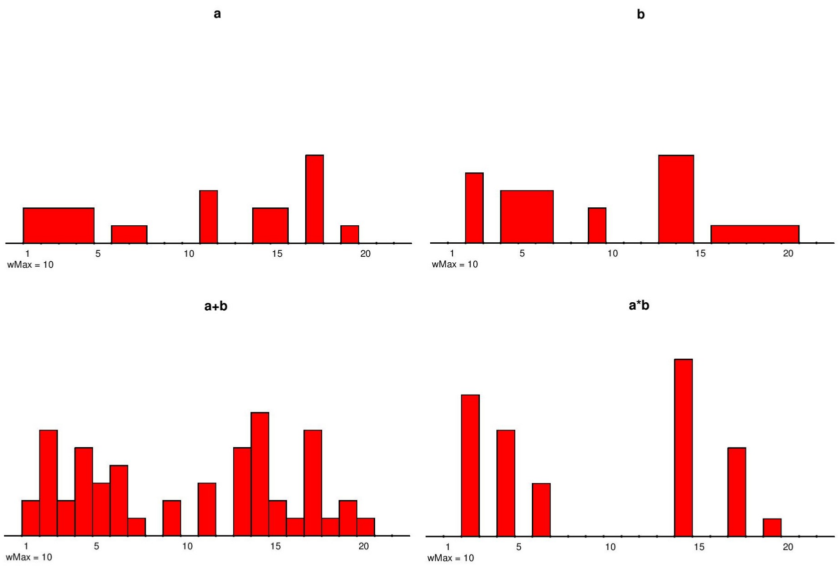
$$T(l(u, v)) \subseteq T(u) \cap T(v).$$

Temporal quantity a with the activity set $T_a \subseteq \mathcal{T}$ describes the changes of properties of nodes and links:

$$a = \begin{array}{ll} a'(t) & t \in T_a \\ undefined & t \in \mathcal{T} \backslash T_a \end{array}$$

In temporal network we assign temporal quantities to nodes and links.

Temporal quantities allow longitudional approach instead of time slices.



Core maintenance
The problem of maintaining core numbers for a temporal network.

1.2 Generalized core

Node property function: $p(v, \mathcal{C}); v \in \mathcal{V}, \mathcal{C} \subseteq \mathcal{V}, p : L \longrightarrow \mathbb{R}^+$.

Properties:
$p(v, \mathcal{C})$ local: $p(v, \mathcal{C}) = p(v, N(v, \mathcal{C}))\ \forall v \in \mathcal{V}$
$p(v, \mathcal{C})$ monotone: $\mathcal{C}_1 \subset \mathcal{C}_2 \Rightarrow \forall v \in \mathcal{V} : p(v, \mathcal{C}_1) \leq p(v, \mathcal{C}_2)$.

The subgraph $\mathcal{H} = (\mathcal{C}, \mathcal{L}(\mathcal{C}))$ induced by the set $C \subseteq \mathcal{V}$ is a p-core at level $t \in \mathbb{R}$ iff $\forall v \in \mathcal{C} : t \leq f(v, \mathcal{C})$ and $\mathcal{C}$ is maximal such set.

Examples of node property function

1. $p_1(v, \mathcal{C}) = \deg_{\mathcal{C}}(v)$: node degree within $\mathcal{C}$
2. $p_2(v, \mathcal{C}) = \text{indeg}_{\mathcal{C}}(v) + \text{outdeg}_{\mathcal{C}}(v)$: if lines are directed it holds $p_2 = p_1$
3. $\mathbf{p_3(v, \mathcal{C}) = \sum_{u \in N(v, \mathcal{C})} w(v, u)}$ **for** $\mathbf{w : L \to \mathbb{R}_0^+}$ **: sum of weights of incident lines within** $\mathcal{C}$
4. $p_4(v, \mathcal{C}) = \max_{u \in N(v, \mathcal{C})} w(v, u)$ for $w : L \to \mathbb{R}$: maximal weight of incident lines within $\mathcal{C}$
5. $p_5(v, \mathcal{C}) = \frac{\deg_{\mathcal{C}}(v)}{\deg(v)}$ if $\deg(v) > 0$ else $f_5(v, \mathcal{C}) = 0$: fraction of neighbors within $\mathcal{C}$.
6. $p_6(v, \mathcal{C}) = \frac{\sum_{u \in N(v, \mathcal{C})} w(v,u)}{\sum_{u \in N(v)} w(v,u)}$ for $w : L \to \mathbb{R}_0^+$: fraction of sum of weights of incident lines within $\mathcal{C}$.

2 Related work

Seidman 1983

posplositve na druge lastnosti vozlisc, posplošitve na 2-mode
ostali algoritmi za core maintenance

3 Algorithm

Core decomposition

```
CoreDecomposition(𝒩):
C = V
k = 1
while C ≠ ∅:
        while ∃ u ∈ C ∋: deg(u) < k:
                for v ∈ N(u, C):
                        deg(v) = deg(v) − 1
                C = C\v
                core(u) = k − 1
        k = k + 1
```

Simple algorithm for cores in temporal networks

```
TemporalCores(𝒩):
D = {u: [triples (start, finish, deg)]}
CoreHierarchy = {u: [triples with deg = 0]}
D = (D.filter(deg > 0)).remove(empty triples)
Dmin = {u: min deg}
while D not empty:
        (dmin, u) = (deg, u) ∋: (u, deg) ∈ Dmin ∧ deg is min deg
        core = [triples from D[u] ∋: deg[u] from triple is equal to dmin]
        CoreHierarchy[u].add(core)
        change = core.set(deg = −1)
        D[u] = D[u].add(change).cutAt(dmin) \\ value >= dmin
        for l in 𝒩.star(u):
                v = other end−node of l
                if not v in D: continue
                changeLink = l.intersection(change).set(deg = −1)
                if changeLink empty: continue
                diff = D[v].add(changeLink).cutAt(0) \\ value >= 0
                D[v] = diff.set(max(currentValue, dmin))
                if D[v] is empty:
                        delete D[v], Dmin[v]
                else:
                        Dmin[v] = triple ∈ D[v] with min deg
        if D[u] empty:
                delete D[u], Dmin[u]
        else:
                Dmin[u] = triple ∈ D[u] with min deg
return CoreHierarchy
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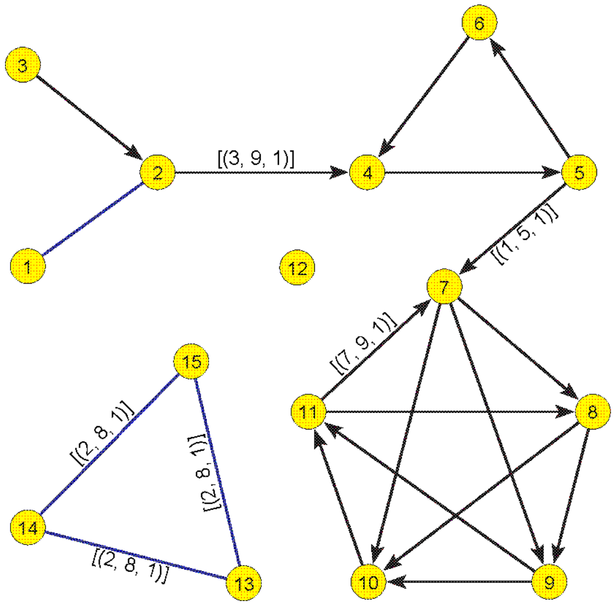
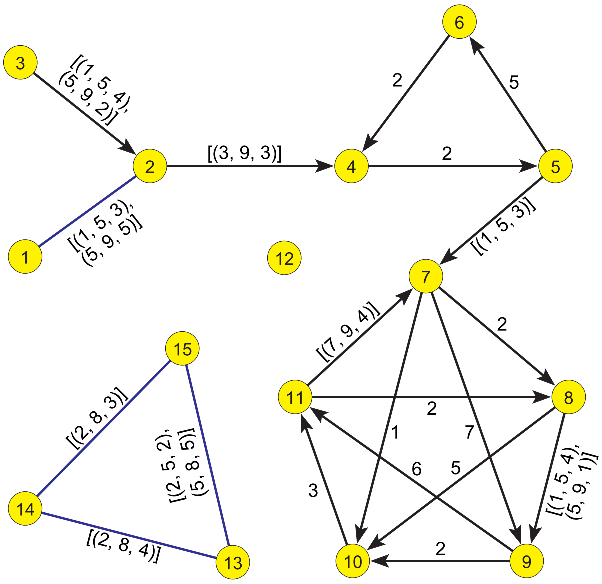
4 Results

4.1 Artificial example



Node	Degree	Core number
1	(1, 9, 1)	(1, 9, 1)
2	(1, 3, 2), (3, 9, 3)	(1, 9, 1)
3	(1, 9, 1)	(3, 9, 1)
4	(1, 3, 2), (3, 9, 3)	(1, 9, 2)
5	(1, 5, 3), (5, 9, 2)	(1, 9, 2)
6	(1, 9, 2)	(1, 9, 2)
7	(1, 5, 4), (5, 7, 3), (7, 9, 4)	(1, 7, 3), (7, 9, 4)
8	(1, 9, 4)	(1, 7, 3), (7, 9, 4)
9	(1, 9, 4)	(1, 7, 3), (7, 9, 4)
10	(1, 9, 4)	(1, 7, 3), (7, 9, 4)
11	(1, 7, 3), (7, 9, 4)	(1, 7, 3), (7, 9, 4)
12	(1, 9, 0)	(1, 9, 0)
13	(1, 2, 0), (2, 8, 2), (8, 9, 0)	(1, 2, 0), (2, 8, 2), (8, 9, 0)
14	(1, 2, 0), (2, 8, 2), (8, 9, 0)	(1, 2, 0), (2, 8, 2), (8, 9, 0)
15	(1, 2, 0), (2, 8, 2), (8, 9, 0)	(1, 2, 0), (2, 8, 2), (8, 9, 0)



Node	Degree	Core number
1	(1, 5, 3), (5, 9, 5)	(1, 5, 3), (5, 9, 5)
2	(1, 3, 7), (3, 9, 10)	(1, 5, 4), (5, 9, 5)
3	(1, 5, 4), (5, 9, 2)	(1, 5, 4), (5, 9, 2)
4	(1, 3, 4), (3, 9, 7)	(1, 5, 4), (5, 9, 5)
5	(1, 5, 10), (5, 9, 7)	(1, 9, 5)
6	(1, 9, 7)	(1, 9, 5)
7	(1, 5, 13), (5, 7, 10), (7, 9, 14)	(1, 9, 10)
8	(1, 5, 13), (5, 9, 10)	(1, 9, 10)
9	(1, 5, 19), (5, 9, 16)	(1, 9, 10)
10	(1, 9, 11)	(1, 9, 10)
11	(1, 7, 11), (7, 9, 15)	(1, 9, 10)
12	(1, 9, 0)	(1, 9, 0)
13	(1, 2, 0), (2, 5, 6), (5, 8, 9), (8, 9, 0)	(1, 2, 0), (2, 5, 5), (5, 8, 7), (8, 9, 0)
14	(1, 2, 0), (2, 8, 7), (8, 9, 0)	(1, 2, 0), (2, 5, 5), (5, 8, 7), (8, 9, 0)
15	(1, 2, 0), (2, 5, 5), (5, 8, 8), (8, 9, 0)	(1, 2, 0), (2, 5, 5), (5, 8, 7), (8, 9, 0)

4.2 Reuters terror news network

[1] Real-life example

Obtained from the CRA (Centering Resonance Analysis) networks produced by Steve Corman and Kevin Dooley at Arizona State University.

Based on all the stories released during 66 consecutive days by the news agency Reuters concerning the September 11 attack on the U.S., beginning at 9:00 AM EST 9/11/01.

Nodes: important words (terms), n = 13332

Links: two nodes appear in the same utterance, m = 243447, undirected, weight is equal to the frequency of appearance, 50859 of them have the weight larger than 1. No loops.

Example: induced subnetwork on 50 most active nodes.

[1] Data available at: http://vlado.fmf.uni-lj.si/pub/networks/data/CRA/terror.htm

Node	Degree
1	(1, 2, 5), (2, 3, 6), (3, 4, 3), (4, 5, 5), (5, 6, 4), (6, 8, 3), (8, 10, 5), (10, 11, 3), (11, 13, 2), (13, 16, 3), (16, 17, 4), (17, 18, 5), (18, 19, 3), (19, 21, 1), (21, 22, 2), (22, 23, 1), (23, 24, 4), (24, 25, 1), (25, 29, 3), (29, 31, 2), (31, 33, 3), (33, 34, 1), (34, 36, 3), (36, 37, 2), (37, 39, 3), (39, 40, 4), (40, 41, 2), (41, 42, 0), (42, 43, 3), (43, 44, 2), (44, 45, 3), (45, 46, 1), (46, 47, 2), (47, 48, 3), (48, 49, 0), (49, 50, 4), (50, 51, 1), (51, 52, 2), (52, 53, 1), (53, 54, 0), (54, 58, 2), (58, 59, 3), (59, 60, 2), (60, 61, 4), (61, 62, 0), (62, 64, 2), (64, 65, 1), (65, 67, 2)
2	(1, 2, 27), (2, 3, 29), ..., (63, 64, 2), (64, 65, 0), (66, 67, 0)
...	
50	(1, 2, 3), (2, 3, 2), (3, 5, 1), (5, 8, 0), (8, 10, 1), (10, 11, 2), (11, 12, 1), (12, 15, 0), (15, 16, 3), (16, 17, 1), (17, 19, 0), (19, 20, 1), (20, 21, 2), (21, 22, 0), (22, 24, 1), (24, 26, 0), (26, 27, 2), (27, 28, 0), (28, 29, 1), (29, 31, 0), (31, 32, 1), (32, 33, 0), (33, 35, 1), (35, 37, 0), (37, 38, 1), (38, 42, 0), (43, 44, 2), (44, 49, 0), (49, 50, 2), (51, 57, 0), (58, 61, 0), (61, 62, 1), (62, 67, 0)

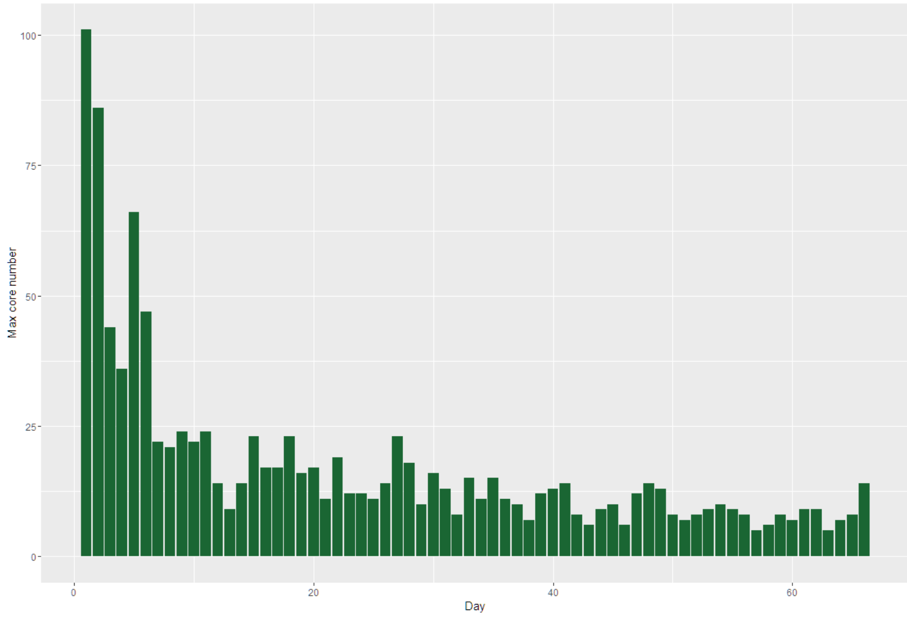
Node	Core number
1	(1, 2, 4), (2, 3, 5), (3, 5, 3), (5, 6, 4), (6, 8, 3), (8, 10, 4), (10, 11, 3), (11, 14, 2), (14, 18, 3), (18, 19, 2), (19, 21, 1), (21, 22, 2), (22, 23, 1), (23, 24, 3), (24, 25, 1), (25, 28, 2), (28, 29, 3), (29, 33, 2), (33, 34, 1), (34, 38, 2), (38, 39, 3), (39, 41, 2), (41, 42, 0), (42, 45, 2), (45, 46, 1), (46, 47, 2), (47, 48, 3), (48, 49, 0), (49, 50, 3), (50, 51, 1), (51, 52, 2), (52, 53, 1), (53, 54, 0), (54, 57, 2), (57, 58, 1), (58, 59, 2), (59, 60, 1), (60, 61, 2), (61, 62, 0), (62, 64, 2), (64, 65, 1), (65, 67, 2)
2	(1, 3, 5), (3, 6, 4), (6, 7, 5), ..., (63, 64, 1), (64, 65, 0), (66, 67, 0)
...	
50	(1, 3, 2), (3, 5, 1), (5, 8, 0), (8, 10, 1), (10, 11, 2), (11, 12, 1), (12, 15, 0), (15, 16, 3), (16, 17, 1), (17, 19, 0), (19, 20, 1), (20, 21, 2), (21, 22, 0), (22, 24, 1), (24, 26, 0), (26, 27, 1), (27, 28, 0), (28, 29, 1), (29, 31, 0), (31, 32, 1), (32, 33, 0), (33, 35, 1), (35, 37, 0), (37, 38, 1), (38, 42, 0), (43, 44, 1), (44, 49, 0), (49, 50, 2), (51, 57, 0), (58, 61, 0), (61, 62, 1), (62, 67, 0)

Temporal cores of order at least 3 appear in the first 11 days and on 30th day

	Node	**Core number** (≥ 3)
25	world	(1, 3, 5), (3, 10, 4)
2	attack	(1, 3, 5), (3, 6, 4), (6, 7, 5), (7, 10, 4), (11, 12, 4), (30, 31, 4)
9	washington	(1, 3, 5), (3, 6, 4), (6, 7, 5), (7, 10, 4), (11, 12, 4)
14	world_trade_ctr	(1, 3, 5), (3, 6, 4), (6, 7, 5), (30, 31, 4)
4	people	(1, 3, 5), (3, 6, 4), (6, 7, 5), (7, 8, 4)
21	pentagon	(1, 3, 5), (3, 4, 4), (5, 6, 4), (6, 7, 5)
7	new_york	(1, 3, 5), (3, 6, 4), (6, 7, 5), (30, 31, 4)
8	pres_bush	(1, 3, 5), (3, 6, 4), (6, 7, 5), (7, 10, 4), (11, 12, 4)
10	official	(1, 3, 5), (3, 4, 4), (5, 6, 4), (6, 7, 5)
43	tower	(1, 3, 5), (3, 4, 4), (6, 7, 5)
34	time	(1, 3, 5), (3, 4, 4), (5, 6, 4), (7, 8, 4)
18	city	(1, 3, 5), (3, 4, 4)
20	tuesday	(1, 3, 5), (3, 7, 4)
13	plane	(1, 3, 5), (3, 7, 4)
15	security	(1, 2, 4), (2, 3, 5), (5, 6, 4)
1	united_states	(1, 2, 4), (2, 3, 5), (5, 6, 4), (8, 10, 4)
19	war	(1, 2, 4), (2, 3, 5), (5, 8, 4)
29	worker	(1, 2, 4), (2, 3, 5)
47	wednesday	(2, 3, 5), (3, 4, 4), (8, 10, 4)
12	military	(1, 2, 4), (5, 6, 4), (30, 31, 4)
5	afghanistan	(1, 3, 4), (5, 6, 4), (6, 7, 5), (8, 10, 4), (30, 31, 4)
6	bin_laden	(1, 4, 4), (5, 6, 4), (6, 7, 5), (7, 10, 4), (11, 12, 4)
36	strike	(2, 3, 4), (5, 6, 4), (6, 7, 5), (30, 31, 4)
28	week	(5, 6, 4), (6, 7, 5), (8, 10, 4), (11, 12, 4)
48	nation	(1, 3, 4), (5, 6, 4)
40	terrorist	(1, 3, 4), (6, 7, 4)
17	country	(1, 3, 4), (5, 10, 4)
23	government	(1, 3, 4), (5, 6, 4)
30	office	(1, 3, 4)
24	leader	(1, 4, 4), (6, 10, 4)
49	police	(2, 4, 4), (5, 6, 4)
31	group	(2, 3, 4), (6, 7, 4)
42	pakistan	(2, 3, 4), (5, 7, 4)
32	air	(2, 3, 4), (5, 6, 4)
27	day	(2, 3, 4), (5, 6, 4)
35	hijack	(2, 3, 4)
26	terrorism	(2, 3, 4)
38	flight	(2, 3, 4)
39	tell	(2, 3, 4)
16	american	(2, 3, 4)
41	airport	(2, 3, 4)
45	new	(2, 3, 4)
22	force	(5, 6, 4)

	Node	p_S-core number (≥ 20)
2	attack	(1, 3, 86), (3, 4, 44), (4, 5, 36), (5, 6, 66), (6, 7, 47), (7, 8, 22), (8, 9, 21), (9, 10, 24), (10, 11, 22), (11, 12, 24), (15, 16, 23), (18, 19, 20), (27, 28, 23)
7	new_york	(1, 2, 101), (2, 3, 86), (3, 4, 42), (4, 5, 35), (5, 6, 66), (6, 7, 47), (8, 9, 21), (9, 10, 24), (10, 11, 22), (11, 12, 24), (15, 16, 23), (18, 19, 20)
14	world_trade_c	(1, 2, 101), (2, 3, 86), (3, 4, 44), (4, 5, 35), (5, 6, 66), (6, 7, 47), (8, 9, 20), (9, 10, 21), (10, 11, 22), (11, 12, 24), (15, 16, 23), (18, 19, 20)
9	washington	(1, 2, 80), (2, 3, 61), (3, 4, 27), (4, 5, 28), (5, 6, 66), (6, 7, 47), (8, 9, 21), (9, 10, 24), (10, 11, 22), (11, 12, 24), (15, 16, 23), (18, 19, 20)
21	pentagon	(1, 3, 86), (3, 4, 44), (4, 5, 32), (5, 6, 66), (6, 7, 47), (8, 9, 20), (9, 10, 21), (10, 11, 22), (11, 12, 24), (15, 16, 23), (18, 19, 20)
1	united_states	(1, 2, 86), (2, 3, 71), (3, 4, 34), (4, 5, 29), (5, 6, 50), (6, 7, 47), (7, 8, 22), (15, 16, 23), (18, 19, 23), (27, 28, 23)
28	week	(5, 6, 35), (6, 7, 27), (7, 8, 22), (8, 9, 21), (9, 10, 24), (10, 11, 22), (11, 12, 24)
4	people	(1, 2, 48), (2, 3, 52), (3, 4, 28), (4, 5, 32), (5, 6, 29), (6, 7, 34), (18, 19, 20)
12	military	(1, 2, 25), (2, 3, 42), (5, 6, 26), (15, 16, 23), (18, 19, 23), (27, 28, 23)
5	afghanistan	(1, 2, 22), (2, 3, 28), (5, 6, 29), (6, 7, 21), (15, 16, 23), (27, 28, 23)
6	bin_laden	(1, 2, 22), (2, 3, 28), (3, 4, 20), (5, 6, 29), (6, 7, 21), (18, 19, 20)
10	official	(1, 2, 40), (2, 3, 54), (3, 4, 34), (5, 6, 29), (6, 7, 36), (18, 19, 23)
43	tower	(1, 2, 101), (2, 3, 72), (3, 4, 41), (4, 5, 32), (5, 6, 38), (6, 7, 32)
35	hijack	(1, 2, 67), (2, 3, 86), (3, 4, 44), (4, 5, 28), (5, 6, 50), (6, 7, 34)
13	plane	(1, 3, 86), (3, 4, 44), (4, 5, 32), (5, 6, 50), (6, 7, 34)
20	tuesday	(1, 3, 86), (3, 4, 44), (4, 5, 36), (5, 6, 66), (6, 7, 47)
3	taliban	(2, 3, 28), (6, 7, 20), (15, 16, 23), (27, 28, 23)
36	strike	(2, 3, 29), (5, 6, 29), (18, 19, 22), (27, 28, 23)
17	country	(1, 2, 24), (2, 3, 31), (5, 6, 26), (18, 19, 20)
8	pres_bush	(1, 2, 48), (2, 3, 44), (5, 6, 29), (6, 7, 21)
41	airport	(1, 2, 25), (2, 3, 44), (4, 5, 25), (5, 6, 24)
15	security	(1, 2, 25), (2, 3, 30), (5, 6, 24)
16	american	(1, 2, 48), (2, 3, 30), (5, 7, 20)
18	city	(1, 2, 60), (2, 3, 52), (3, 4, 22)
25	world	(1, 2, 34), (2, 3, 44), (18, 19, 20)
27	day	(1, 2, 21), (2, 3, 36), (5, 6, 20)
32	air	(2, 3, 34), (5, 6, 29), (27, 28, 23)
38	flight	(1, 2, 25), (2, 3, 52), (4, 5, 20)
48	nation	(1, 2, 31), (2, 3, 38), (5, 6, 23)
40	terrorist	(1, 2, 40), (2, 3, 29)
19	war	(2, 3, 34), (5, 6, 29)
23	government	(1, 2, 28), (2, 3, 36)
46	buildng	(1, 2, 34), (2, 3, 44)
30	office	(1, 2, 34), (2, 3, 20)
26	terrorism	(5, 6, 20)
29	worker	(1, 2, 24)
31	group	(2, 3, 26)
34	time	(2, 3, 36)
22	force	(5, 6, 26)
24	leader	(1, 2, 22)
42	pakistan	(5, 6, 29)
44	bomb	(1, 2, 23)
45	new	(2, 3, 30)
47	wednesday	(2, 3, 52)
49	police	(2, 3, 20)

Max p_S-core numbers by days from the event



4.3 Violence network

[2] Real-life example

Roberto Franzosi collected from the journal news in the period January 1919 - December 1922 information about the different types of interactions between political parties and other groups of people in Italy. The violence network contains only the data about violent actions and counts the number of interactions per month.

Nodes: groups of people, n = 29

Links: violent interactions, m = 105

Node		**Core number** (≥ 3)
16	workers	(29, 30, 3), (33, 34, 3), (39, 41, 3)
1	undefined	(29, 30, 3), (39, 40, 3)
2	?	(31, 32, 3), (33, 34, 3), (40, 41, 3)
3	people	(31, 32, 3), (33, 34, 3), (39, 40, 3)
4	police	(31, 32, 3), (33, 34, 3), (40, 41, 3)
21	catholics	(33, 34, 3)
7	fascists	(29, 30, 3), (31, 32, 3), (33, 34, 3), (39, 41, 3)
8	communists	(29, 30, 3)
10	socialists	(31, 32, 3), (40, 41, 3)

[2]Franzosi, R., 1997. Mobilization and CounterMobilization Processes: From the Red Years (1919-20) to the Black Years (1921-22) in Italy. A New Methodological Approach to the Study of Narrative Data. Theory and Society, 26(2-3), 275-304

Node		Core number (≥ 2)
1	undefined	(15, 16, 2), (17, 18, 2), (25, 29, 2), (29, 30, 3), (31, 32, 2), (38, 39, 2), (39, 40, 3), (41, 44, 2), (45, 46, 2), (48, 49, 2)
2	?	(14, 16, 2), (17, 18, 2), (28, 29, 2), (31, 32, 3), (32, 33, 2), (33, 34, 3), (34, 35, 2), (40, 41, 3)
3	people	(16, 18, 2), (23, 24, 2), (25, 26, 2), (28, 30, 2), (31, 32, 3), (33, 34, 3), (35, 37, 2), (39, 40, 3), (41, 43, 2), (48, 49, 2)
4	police	(11, 12, 2), (14, 20, 2), (21, 23, 2), (29, 31, 2), (31, 32, 3), (32, 33, 2), (33, 34, 3), (34, 37, 2), (38, 40, 2), (40, 41, 3)
5	land owners	(15, 16, 2), (17, 20, 2), (29, 30, 2), (36, 37, 2), (38, 40, 2), (42, 43, 2)
7	fascists	(11, 12, 2), (16, 17, 2), (19, 20, 2), (21, 24, 2), (25, 29, 2), (29, 30, 3), (30, 31, 2), (31, 32, 3), (32, 33, 2), (33, 34, 3), (34, 37, 2), (38, 39, 2), (39, 41, 3), (41, 44, 2), (45, 46, 2), (48, 49, 2)
8	communists	(28, 29, 2), (29, 30, 3), (31, 33, 2), (35, 37, 2), (43, 44, 2)
9	workers (agr)	(15, 16, 2), (17, 20, 2), (28, 30, 2), (31, 32, 2), (33, 35, 2), (38, 43, 2), (45, 46, 2)
10	socialists	(11, 12, 2), (16, 18, 2), (19, 20, 2), (22, 23, 2), (25, 26, 2), (27, 30, 2), (31, 32, 3), (33, 37, 2), (38, 40, 2), (40, 41, 3), (41, 42, 2)
12	war affected	(35, 36, 2), (39, 40, 2)
13	protesters	(15, 16, 2), (21, 22, 2), (29, 30, 2), (31, 32, 2), (38, 40, 2)
16	workers	(11, 12, 2), (14, 18, 2), (19, 20, 2), (21, 24, 2), (25, 26, 2), (27, 29, 2), (29, 30, 3), (30, 33, 2), (33, 34, 3), (34, 37, 2), (38, 39, 2), (39, 41, 3), (41, 44, 2), (45, 46, 2)
17	the right	(17, 18, 2), (41, 42, 2)
19	populars	(41, 42, 2)
20	students	(17, 18, 2)
21	catholics	(33, 34, 3)
25	republicans	(26, 27, 2)
26	thugs	(29, 30, 2)
27	prisoners/arrested	(40, 41, 2)

Node		p_S**-ore number** (≥ 10)
16	workers	(1, 2, 27), (10, 11, 11), (14, 15, 27), (16, 17, 11), (17, 18, 17), (18, 19, 12), (22, 23, 17), (25, 26, 11), (27, 28, 18), (28, 29, 16), (29, 30, 53), (30, 31, 56), (31, 32, 51), (32, 33, 30), (33, 34, 17), (34, 35, 71), (35, 36, 76), (36, 37, 53), (37, 38, 11), (38, 39, 23), (39, 40, 54), (40, 41, 13), (41, 42, 174), (42, 43, 25), (43, 44, 20), (45, 46, 15), (46, 47, 25)
10	socialists	(10, 11, 10), (12, 13, 29), (27, 28, 30), (28, 29, 31), (29, 30, 64), (30, 31, 29), (31, 32, 17), (32, 33, 14), (33, 34, 24), (34, 35, 38), (35, 36, 23), (36, 37, 26), (37, 38, 13), (38, 39, 19), (39, 40, 54), (45, 46, 13)
4	police	(1, 2, 36), (6, 7, 15), (10, 11, 24), (12, 13, 29), (14, 15, 27), (15, 16, 13), (16, 17, 24), (17, 18, 17), (18, 19, 12), (22, 23, 17), (31, 32, 17)
7	fascists	(25, 26, 11), (27, 28, 30), (28, 29, 31), (29, 30, 64), (30, 31, 56), (31, 32, 51), (32, 33, 30), (33, 34, 24), (34, 35, 71), (35, 36, 76), (36, 37, 53), (37, 38, 13), (38, 39, 23), (39, 40, 54), (40, 41, 13), (41, 42, 174), (42, 43, 25), (43, 44, 20), (45, 46, 15), (46, 47, 25)
9	workers (agr)	(10, 11, 24), (16, 17, 24), (28, 29, 16), (30, 31, 13), (36, 37, 11), (39, 40, 15), (43, 44, 10)
1	undefined	(25, 26, 11), (27, 28, 12), (28, 29, 16), (41, 42, 133), (45, 46, 11)
8	communists	(29, 30, 13), (30, 31, 10), (31, 32, 12)
13	protesters	(6, 7, 15), (15, 16, 13), (16, 17, 20)
12	war affected	(1, 2, 36)
3	people	(28, 29, 12)

5 Conclusion

Improve the complexity of the algorithm

Extend the algorithm to generalized temporal cores

Find user friendly presentations of results

Compare with the streaming core algorithms

Temporal Quantities - a Python 3 library for temporal network analysis: http://vladowiki.fmf.uni-lj.si/doku.php?id=tq

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