



NA2-5,
statistics

V. Batagelj

Autocorrelation

Network
statistics

CUG

QAP

What else?

Resources

Network Analysis 2

Statistical Approaches and Modeling

Statistics

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Outline

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- 1 Autocorrelation
- 2 Network statistics
- 3 CUG
- 4 QAP
- 5 What else?
- 6 Resources



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Current version of slides (August 24, 2022 at 14 : 47): [slides PDF](#)



Continuous Autocorrelation

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Assume that each node has score on continuous variable, such as age or rank.

Positive autocorrelation exists when nodes of similar value of variable tend to be adjacent

- Friendships tend to be homophilous wrt age
- Mentoring tends to be heterophilous wrt age

We can measure similarity via difference (Geary) or product (Moran).



Autocorrelation Measures

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Two measures were “borrowed” from spatial statistics:

- Geary's C
 - Also called Geary's [Contiguity] Ratio
 - Most sensitive to local autocorrelation
- Moran's I
 - Measures autocorrelation not only on variable values or location (adjacency), but rather on both simultaneously
 - More sensitive to global autocorrelation
- I is about covariation of pairs, C is about variation in variable values



Statnet `nacf`

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Both measures are built in the `statnet's nacf`. It computes dependence statistics for the given vector on network structure, for neighborhoods of various orders. It produces a vector of measures for multiple steps out into the network, up to the theoretical maximum indicated by the order of the network.

For our purposes, we consider only the immediate neighborhood around each node, meaning that we are interested in autocorrelation between nodes that are just one step from one another. The vector goes from 0 steps to however many nodes are present in the network. Thus, the [2] in the scripts refers to the second measure in the vector (one step).



Geary's C

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Let $a_{uv} > 0$ indicate adjacency of nodes u and v , and X_u indicate the score of node u on attribute X (e.g., age)

$$C = (n - 1) \frac{\sum_{u,v} a_{uv} (x_u - x_v)^2}{2 \sum_{u,v} a_{uv} \cdot \sum_u (x_u - \bar{x})^2}$$

Range of values: $0 \leq C \leq 2$

- $C = 1$ indicates independence;
- $C > 1$ indicates negative autocorrelation;
- $C < 1$ indicates positive autocorrelation (homophily)



Moran's I

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$$I = n \frac{\sum_{u,v} a_{uv}(x_u - \bar{x})(x_v - \bar{x})}{\sum_{u,v} a_{uv} \cdot \sum_u (x_u - \bar{x})^2}$$

Ranges between -1 and +1

- Expected value under independence is $\frac{-1}{n-1}$
- $I \rightarrow +1$ when positive autocorrelation
- $I \rightarrow -1$ when negative autocorrelation



Simple example

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```
library(statnet)
g <- c(
  0, 1, 0, 1, 0, 0, 0, 0, 0,
  0, 0, 1, 0, 1, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 1, 0, 0, 0,
  0, 0, 0, 0, 1, 0, 1, 0, 0,
  0, 0, 0, 0, 0, 1, 0, 1, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 1,
  0, 0, 0, 0, 0, 0, 0, 1, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 1,
  0, 0, 0, 0, 0, 0, 0, 0, 0 )
G <- symmetrize(matrix(g,byrow=TRUE,nrow=9))
rownames(G) <- colnames(G) <- c("A", "B", "C", "D", "E", "F", "G", "H",
plot.sociomatrix(G)
gplot(G,gmode="graph",displaylabels=TRUE)
a1 <- c( 1, 2, 3, 2, 3, 4, 3, 4, 5 )
gplot(G,gmode="graph",displaylabels=TRUE,vertex.cex=a1)
nacf(G,a1,type="geary",mode="graph") [2]
nacf(G,a1,type="morán",mode="graph") [2]
a2 <- c(3, 4, 3, 4, 3, 2, 1, 2, 5 )
nacf(G,a2,type="geary",mode="graph") [2]
nacf(G,a2,type="morán",mode="graph") [2]
a3 <- c(4, 1, 4, 2, 5, 2, 3, 3, 3 )
nacf(G,a3,type="geary",mode="graph") [2]
nacf(G,a3,type="morán",mode="graph") [2]
```




Positive Autocorrelation

Similar adjacent; Geary's $C < 1$, Moran's $I > -0.125$

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Autocorrelation

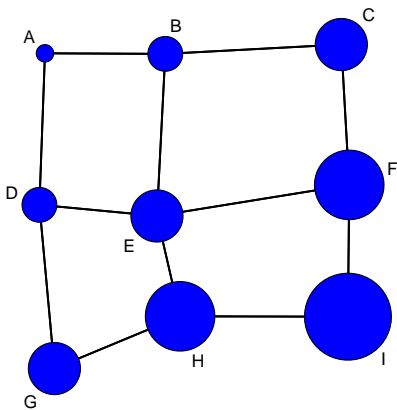
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Node	Attrib
A	1
B	2
C	3
D	2
E	3
F	4
G	3
H	4
I	5

Geary's C : 0.333

Moran's I : 0.500



No Autocorrelation

Random pattern; Geary's $C \approx 1$, Moran's $I \approx -0.125$

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Autocorrelation

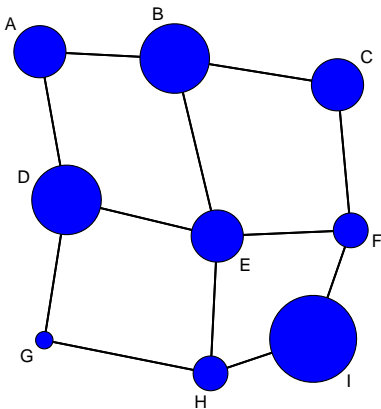
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Resources



Node	Attrib
A	3
B	4
C	3
D	4
E	3
F	2
G	1
H	2
I	5

Geary's C : 1.000

Moran's I : -0.250



Negative Autocorrelation

Dissimilars adjacent; Geary's $C > 1$, Moran's $I < -0.125$

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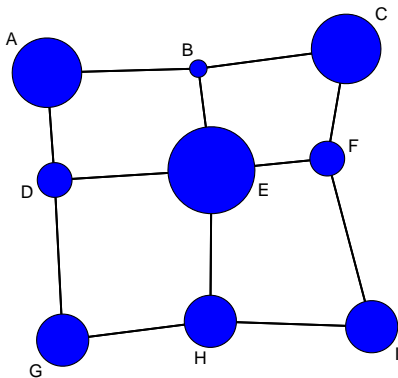
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Resources



Node	Attrib
A	4
B	1
C	4
D	2
E	5
F	2
G	3
H	3
I	3

Geary's C : 1.833

Moran's I : -0.875



Interpreting Autocorrelation

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Moran's I ranges from -1 to 1 and is interpreted like Pearson's Correlation Coefficient.

- A value near $+1.0$ indicates clustering (adjacency tends to accompany similarity along a dimension)
- A value near -1.0 indicates dispersion (adjacency tends to accompany dissimilarity along a dimension)
- a value near 0 indicates random distribution (independence)

Geary's C ranges from 0 to 2 . Just replace $+1$, -1 and 0 in the above with 0 , 2 and 1 .



Florentine families

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Resources

What is the dependence of the wealth of a family on marriage network?

```
> library(statnet)
> data(florentine)
> fw <- flomarriage %v% "wealth"
> gplot(flomarriage, displaylabels=TRUE, vertex.cex=0.025*fw,
+ gmode="graph", main='Florentine families')
> I <- nacf(flomarriage, fw, type="moran", mode="graph")
> I
      0          1          2          3          4
1.00000000 -0.31073529  0.06531299 -0.06045322 -0.06267282  0.0
> C <- nacf(flomarriage, fw, type="geary", mode="graph")
> C
      0          1          2          3          4
0.00000000  1.683607336  0.811642725  0.899673648  0.826831324  0.00
> I[2]
      0          1
-0.3107353
> C[2]
      1
1.683607
```

We can see that the Florentine marriage network is (moderately, $I=0.31$) negatively ($C = 1.68$) autocorrelated with respect to the family wealth. Usually bride and spouse were not both equally rich.



Problem with network statistics

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Resources

Some issues with network statistics:

- Samples non-random
- Often work with populations
- Observations not independent
- Distributions unknown

One crucial thing in SNA is that even when networks are completely 'random' they exhibit certain non-random network features. For example, the embeddedness in triads. When a random network is dense, nodes will have a higher chance to be embedded in triads by default. Hence, you cannot really say that in one network nodes are more clustered than in another if you do not consider this.



General strategy

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Network inference assuming conditional independence:

- 1 Calculate on the “observed” network a network statistic (global measure) that you are interested in.
- 2 Think about the properties of the network that you want to conserve.
- 3 Generate many “random” networks that have the same properties as the “observed” network.
- 4 Calculate the network statistic on these “conditional random networks” and compare this baseline distribution against the actually observed network statistic in the “observed” network.



Conditional Uniform Graphs (CUG)

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Resources

This strategy is implemented in the `sna` function `cug.test` – Conditional uniform graphs (CUGs).

It supports conditioning the simulated networks on three of the possible modes (`cmode`): *size*; *number of edges*; and the *distribution of dyads*. Select the conditioning mode according to what you suspect about your own network of interest. (Note: CUG can be made to condition on other properties.)

In the following example we will try for comparison all three options. For a statistics we will use the `betweenness` centralization. Centralization involves two items: the network being analyzed; and the name of the centrality measure being applied. Other global measures will, therefore, not require the `FUN.arg=list()`, argument.



Conditional Uniform Graphs (CUG)

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Resources

```
library(statnet)
data(florentine)
gplot(flobusiness, gmode="graph", displaylabels=TRUE)

rSize <- cug.test(flobusiness, centralization,
  FUN.arg=list(FUN=betweenness), mode="graph", cmode="size")
rEdges <- cug.test(flobusiness, centralization,
  FUN.arg=list(FUN=betweenness), mode="graph", cmode="edges")
rDyad <- cug.test(flobusiness, centralization,
  FUN.arg=list(FUN=betweenness), mode="graph", cmode="dyad.census")

# Aggregate results
Betweenness <- c(rSize$obs.stat, rEdges$obs.stat, rDyad$obs.stat)
PctGreater <- c(rSize$pgteobs, rEdges$pgteobs, rDyad$pgteobs)
PctLess <- c(rSize$plteobs, rEdges$plteobs, rDyad$plteobs)
report <- cbind(Betweenness, PctGreater, PctLess)
rownames(report) <- c("Size", "Edges", "Dyads")
report
```



Conditional Uniform Graphs (CUG)

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	Betweenness	PctGreater	PctLess
Size	0.2057143	0.001	0.999
Edges	0.2057143	0.713	0.289
Dyads	0.2057143	0.738	0.263

Autocorrelation

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What else?

Resources

Consider the visualization of the network. Given the network's structure, it is at least somewhat dominated by the Barbadori and Medici families (betweenness centralization = 0.21). But is that level of centralization special to the Florentine business network, or is this something that we would normally expect for a network this size? Is it something that we would normally expect for a network with this number of edges? Is it something that we would normally expect for a network with this distribution of dyads?

As we can see in the output above, this level of centralization is very uncommon in a network of this size. But it is not at all uncommon in a network with the same number of edges, or the same distribution of dyads.



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We can depict the same information graphically by displaying the distribution of betweenness centralization measures for the randomly generated networks, and indicating where the betweenness centralization measure of 0.21 lies in comparison to each distribution.

```
par(mfrow=c(1,3))
plot(rSize,main="Betweenness \nConditioned on Size")
plot(rEdges,main="Betweenness \nConditioned on Edges")
plot(rDyad,main="Betweenness \nConditioned on Dyads")
par(mfrow=c(1,1))
```



Conditional Uniform Graphs (CUG)

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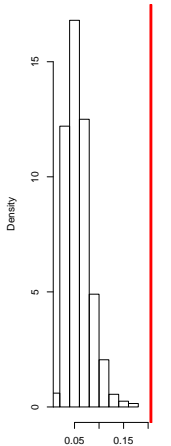
CUG

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What else?

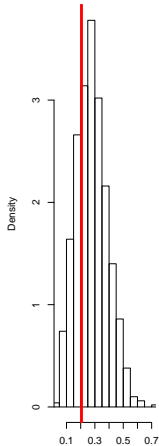
Resources

**Betweenness
Conditioned on Size**



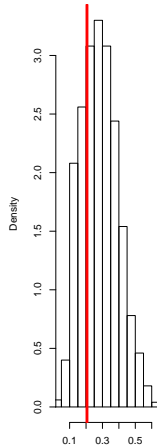
CUG Replicates
Conditioning: size Repls: 1000

**Betweenness
Conditioned on Edges**



CUG Replicates
Conditioning: edges Repls: 1000

**Betweenness
Conditioned on Dyads**



CUG Replicates
Conditioning: dyad.census Repls: 10



Quadratic Assignment Procedure (QAP)

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Resources

Quadratic assignment procedure (QAP) is similar to CUG, in that it uses simulation in order to generate a distribution of hypothetical networks. But QAP controls for network structure, as compared with CUG, which controls for size, the number of edges, or dyad census. QAP is useful for running a variety of statistics. It is based on the *permutation test*

- Get observed test statistic
- Construct a distribution of test statistics under null hypothesis
 - Thousands of permutations of actual data
- Count proportion of statistics on permuted data that are as large as the observed
 - This is the p-value of the test



Quadratic Assignment Procedure (QAP) graph correlation

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Resources

Did the Florentine families base their business dealings on marriage ties? (Or maybe their marriages are based on their business ties?)

Get the **graph correlation value**

```
> gcor(flobusiness, flomarriage)
[1] 0.3718679
```

Marriage and business ties are moderately correlated ($r=0.37$).

Is it significant?

```
> (rCor <- qaptest(list(flobusiness, flomarriage), gcor,
+   g1=1, g2=2, reps=1000))
QAP Test Results
```

```
Estimated p-values:
  p(f(perm) >= f(d)) : 0.001
  p(f(perm) <= f(d)) : 1
```

The correlation is significant at the 0.05 alpha level. We know this because less than 5% of the permuted networks - or in this case, all of them - exhibited correlation coefficients that were less than that of the value we calculated for the marriage network and the business.

```
> plot(rCor, xlim=c(-0.25, 0.4))
```





Quadratic Assignment Procedure (QAP) graph correlation

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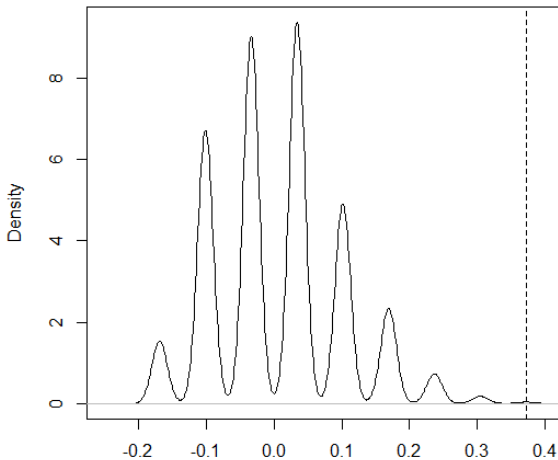
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What else?

Resources

Estimated Density of QAP Replications



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What else?

Resources

- QAP and regression
- MRQAP
- ERGM – Exponential random graph models
- Stochastic actor-oriented models, Siena
- Stochastic blockmodeling



Resources I

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Autocorrelation








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Resources

-  Decker, Krackhardt, Snijders: Sensitivity of MRQAP Tests to Collinearity and Autocorrelation Conditions.
-  Grund, Thomas: Lectures, 2016.
-  Kilduff, Martin, Tsai, Wenpin: Social Networks and Organizations. SAGE publications, 2003.
-  Kolaczyk, Eric D., Csárdi, Gábor: Statistical Analysis of Network Data with R (Use R!). Springer, 2014.
-  Lusher, Dean, Koskinen, Johan, Robins, Garry : Exponential Random Graph Models for Social Networks: Theory, Methods, and Applications. Cambridge University Press, 2013.
-  Murphy, Phil: Practicum 9 - Hypothesis Testing in Network Analysis. 2017.
-  Simpson, William: "QAP – the Quadratic Assignment Procedure" , Harvard Business School, 2001. page