

Rnet, cohesion

V. Batagelj

Islands

Cores

Generalized cores

Network Analysis

Structure of networks: cohesion

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IMFM Ljubljana, IAM UP Koper and NRU HSE Moscow

Master's programme

Applied Statistics with Social Network Analysis International Laboratory for Applied Network Research NRU HSE, Moscow 2020

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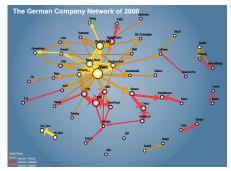
Outline



Cores

Generalized cores

- Islands
 Cores
- 3 Generalized cores



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L. Krempl, MPI.

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Current version of slides (November 16, 2020 at 00:06): slides PDF

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Islands

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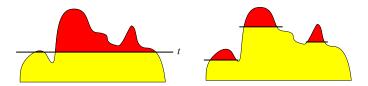
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Islands

Cores

Generalized cores

If we represent a given or computed value of nodes / links as a height of nodes / links and we immerse the network into a water up to selected level we get *islands*. Varying the level we get different islands.



We developed very efficient algorithms to determine the islands hierarchy and to list all the islands of selected sizes. See details.

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... Islands

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Islands

Cores

Generalized cores Islands are very general and efficient approach to determine the 'important' subnetworks in a given network.

We have to express the goals of our analysis with a related property of the nodes or weight of the links. Using this property we determine the islands of an appropriate size (in the interval kto K).

In large networks we can get many islands which we have to inspect individually and interpret their content.

An important property of the islands is that they identify locally important subnetworks on different levels. Therefore they detect also emerging groups.

The set of nodes $C \subseteq V$ is a local node peak, if it is a regular node island and all of its nodes have the same value. Node island with a single local node peak is called a simple node island. In similar way we define simple link island.



... Islands

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A set of nodes $C \subseteq \mathcal{V}$ is a *regular node island* in network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, p), p : \mathcal{V} \to \mathbb{R}$ iff it induces a connected subgraph and the nodes from the island are 'higher' than the neighboring nodes

$$\max_{u\in N(C)}p(u)<\min_{v\in c}p(v)$$

A set of nodes $C \subseteq \mathcal{V}$ is a *regular link island* in network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, w), w : \mathcal{L} \to \mathbb{R}$ iff it induces a connected subgraph and the links inside the island are 'stronger related' among them than with the neighboring nodes – in \mathcal{N} there exists a spanning tree \mathcal{T} over C such that

$$\max_{(u,v)\in\mathcal{L}, u\notin C, v\in C} W(u,v) < \min_{(u,v)\in\mathcal{T}} W(u,v)$$

Network/Create Partition/Islands/Line Weights Operations/Network+Vector/Islands/Vertex Property

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Some properties of node islands

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Islands

- Cores
- Generalized cores

- The sets of nodes of connected components of node-cut at selected level *t* are regular node islands.
- The set H_p(N) of all regular node islands of network N is a complete hierarchy:
 - two islands are disjoint or one of them is a subset of the other
 - each node belongs to at least one island
- Node islands are invariant for the strictly increasing transformations of the property *p*.
- Two linked nodes cannot belong to two disjoint regular node islands.

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Simple node islands

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Islands

Cores

Generalized cores

- The set of nodes C ⊆ V is a local node peak, if it is a regular node island and all of its nodes have the same value.
- Node island with a single local node peak is called a simple node island.
- The types of node islands:
 - FLAT all nodes have the same value
 - SINGLE island has a single local node peak
 - MULTI island has more than one local node peaks

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• Only the islands of type FLAT or SINGLE are simple islands.



Some properties of link islands

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Islands

- Cores
- Generalized cores

- The sets of nodes of connected components of link-cut at selected level *t* are regular link islands.
- The set H_w(N) of all nondegenerated regular link islands of network N is hierarchy (not necessarily complete):
 - two islands are disjoint or one of them is a subset of the other

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- Link islands are invariant for the strictly increasing transformations of the weight *w*.
- Two linked nodes may belong to two disjoint regular link islands.



Simple link islands

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Islands

- Cores
- Generalized cores

- The set of nodes C ⊆ V is a local link peak, if it is a regular link island and there exists a spanning tree of the corresponding induced network, in which all links have the same value as the link with the largest value.
- Link island with a single local link peak is called a simple link island.
- The types of link islands:
 - FLAT there exists a spanning tree, in which all links have the same value as the link with the largest value.

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- SINGLE island has a single local link peak.
- MULTI island has more than one local link peaks.
- Only the islands of type FLAT or SINGLE are simple islands.



US patents

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Islands

Cores

Generalized cores US patents network (Nber, US Patents) has 3774768 nodes and 16522438 arcs (1 loop). Without the loop it is acyclic. The weight of an arc is the proportion of paths through the arc from some initial node to some terminal node. We determined al (2,90)-islands. The corresponding subnetwork has 470137 nodes, 307472 arcs and for different *k*: $C_2 = 187610$, $C_5 = 8859$, $C_{30} = 101$, $C_{50} = 30$, ... islands. Rolex

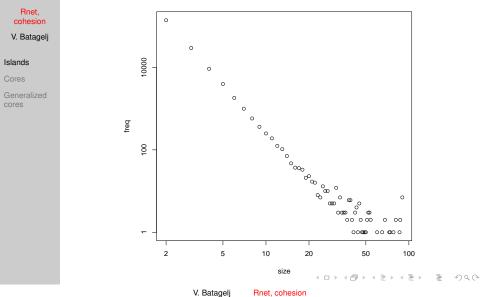
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[41]	1	3	4	1	5	2	1	1	1	1
[51]	2	3	3	2	0	0	0	0	0	1
[61]	0	0	0	0	1	0	0	2	0	0
[71]	0	0	1	1	0	0	0	1	0	0
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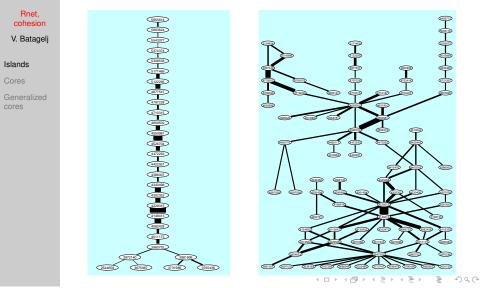


Distribution of island size





Main path and main island in US Patents Nber, US Patents; n = 3774768, m = 16522438



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Main island – Liquid crystal display

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Islands

Table 1: Patents on the liquid-crystal display

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Table 2: Patents on the liquid-crystal display

Table 3: Patents on the liquid-crystal display

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	4558151	Dec 10, 1985	Takatsu, et al. Nematic liquid crystalline compounds
	4583826	Apr 22, 1986	Petrzilka, et al. Phenylethanes
	4621901	Nov 11, 1986	Petrzilka, et al. Novel liquid crystal mintures
	4630856		Petrzika, et al. Benzonitriles
	4657695		Saito, et al. Substituted pyridatines
	4655580	Apr 21, 1987	Fearon, et al. Ethane derivatives
	4695131	Sep 22, 1987	Balkwill, et al. Disubstituted ethanes and their use in liquid
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	4704227	Nov 3, 1987	Krane, et al. Liouid crystal compounds
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	4770503	Sep 13, 1988	Eatenschink, et al. Nitrogen-containing heterocyclic compounds Buchecker, et al. Liquid crystalline compounds
	4795579	Jan 3, 1989	Vanchier, et al. 2.2"-diffnoro-4-alkoxy-4"-hydroxydinhenyds and
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	4951349	Sep 18, 1990	cherc, et al. Active matrix screen for the cour display of television nictures, control system and process for producing
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	5122295	Jun 16, 1992	Weber, et al. Matrix liquid crystal display
	5124824	Jun 23, 1992	Kozaki, et al. Liquid crystal display device comprising a
	1		retardation compensation layer having a maximum principal refractive index in the thickness direction
	5171469	Dec 15, 1992	refractive index in the thickness direction
	5171469		Hittich, et al. Liquid-crystal matrix display Sarawa, et al. Liquid crystal display with ground regions
	5253617	Feb 1, 1994	
	5308538	May 3, 1994	between terminal groups
rs of			Weber, et al. Supertwist liquid-crystal display
ame	5374374	Dec 20, 1994	Weber, et al. Supertwist liquid-crystal display
	5543077	Aug 6, 1996	Rieger, et al. Nematic liquid-crystal composition
	5555116	Sep 10, 1996	Ishikawa, et al. Liquid crystal display having adjacent
			electrode terminals set equal in length
	5683624	Nov 4, 1997	Sekiguchi, et al. Liquid crystal composition
	5855814	Jan 5, 1999	Matsui, et al. Liquid crystal compositions and liquid crystal
			display elements

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Word clouds for LCD island and foam island

Rnet, cohesion

V. Batagelj

Islands

Cores

Generalized cores



Rnet, cohesion

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Rnet.

The Edinburgh Associative Thesaurus

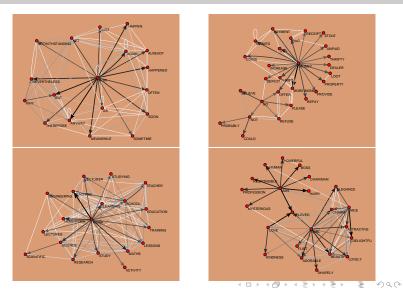
n = 23219, m = 325624, transitivity weight



Islands

Cores

Generalizec cores



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Rnet, cohesion



Dense groups

Rnet, cohesion

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Islands

Cores

Generalized cores Several notions were proposed in attempts to formally describe dense groups in graphs.

Clique of order *k* is a maximal complete subgraph (isomorphic to K_k), $k \ge 3$.

s-plexes, LS sets, lambda sets, cores, ...

For all of them, except for cores, it turned out that they are difficult to detemine.

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Cores and generalized cores

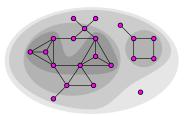
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Cores

Generalized cores



The notion of core was introduced by Seidman in 1983. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a graph. A subgraph $\mathcal{H} = (W, \mathcal{E}|W)$ induced by the set W is a *k*-core or a core of order *k* iff $\forall v \in W$: $\deg_{\mathcal{H}}(v) \geq k$, and \mathcal{H} is a maximal subgraph with this property. The core of maximum order is also called the *main* core.

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The *core number* of node v is the highest order of a core that contains this node. The degree deg(v) can be: in-degree, out-degree, in-degree + out-degree, etc., determining different types of cores.



Properties of cores

Rnet, cohesion

V. Batagelj

Islands

Cores

Generalized cores From the figure, representing 0, 1, 2 and 3 core, we can see the following properties of cores:

- The cores are nested: $i < j \implies \mathcal{H}_j \subseteq \mathcal{H}_i$
- Cores are not necessarily connected subgraphs.

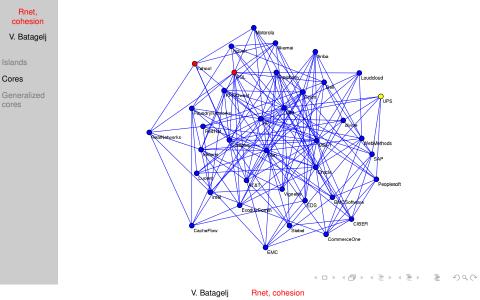
An efficient algorithm for determining the cores hierarchy is based on the following property:

If from a given graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ we recursively delete all nodes, and edges incident with them, of degree less than k, the remaining graph is the k-core.

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Generalized cores

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Cores

Generalized cores

The notion of core can be generalized to networks. Let $\mathcal{N} = (\mathcal{V}, \mathcal{E}, w)$ be a network, where $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a graph and $w : \mathcal{E} \to \mathbb{R}$ is a function assigning values to edges. A *node property function* on **N**, or a *p*-function for short, is a function $p(v, U), v \in \mathcal{V}, U \subseteq \mathcal{V}$ with real values. Let $N_U(v) = N(v) \cap U$. Besides degrees and (corrected) clustering coefficient, here are some examples of *p*-functions:

$$\mathcal{P}_{\mathcal{S}}(v, U) = \sum_{u \in N_U(v)} w(v, u), ext{ where } w : \mathcal{E} o \mathbb{R}^+_0$$

$$p_M(v, U) = \max_{u \in N_U(v)} w(v, u), \text{ where } w : \mathcal{E} \to \mathbb{R}$$

$$p_t(v, \mathcal{U}) = \frac{|\mathcal{L}(\mathcal{U}) \cap \mathcal{L}(K(N^+(v)))|}{|\mathcal{L}(K(N^+(v)))|}$$

 $p_k(v, U) =$ number of cycles of length k through node v in $(U, \mathcal{E}|U)$

The subgraph $\mathcal{H} = (C, \mathcal{E}|C)$ induced by the set $C \subseteq \mathcal{V}$ is a *p*-core at level $t \in \mathbb{R}$ iff $\forall v \in C : t \leq p(v, C)$ and *C* is a maximal such set.

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Additional *p*-functions

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relative density
$$p_{\gamma}(v, C) = \frac{\deg(v, C)}{\max_{u \in N(v)} \deg(u)}$$
, if $\deg(v) > 0$; 0, otherwise

diversity

$$p_{\delta}(v, C) = \max_{u \in N^+(v, C)} \deg(u) - \min_{u \in N^+(v, C)} \deg(u)$$

average weight $p_a(v, C) = \frac{1}{|N(v, C)|} \sum_{u \in N(v, C)} w(v, u)$, if $N(v, C) \neq \emptyset$; 0, otherwise

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Generalized cores algorithms

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Cores

Generalized cores

The function *p* is *monotone* iff it has the property

$$C_1 \subset C_2 \Rightarrow \forall v \in \mathcal{V} : (p(v, C_1) \leq p(v, C_2))$$

The degrees and the functions p_S , p_M and p_k are monotone. For a monotone function the *p*-core at level *t* can be determined, as in the ordinary case, by successively deleting nodes with value of *p* lower than *t*; and the cores on different levels are nested

$$t_1 < t_2 \Rightarrow \mathcal{H}_{t_2} \subseteq \mathcal{H}_{t_1}$$

The *p*-function is *local* iff $p(v, U) = p(v, N_U(v))$. The degrees, p_S and p_M are local; but p_k is **not** local for $k \ge 4$. For a local *p*-function an $O(m \max(\Delta, \log n))$ algorithm for determining the *p*-core levels exists, assuming that $p(v, N_C(v))$ can be computed in $O(\deg_C(v))$. For details see the paper.

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Cores and generalized cores / Pajek commands

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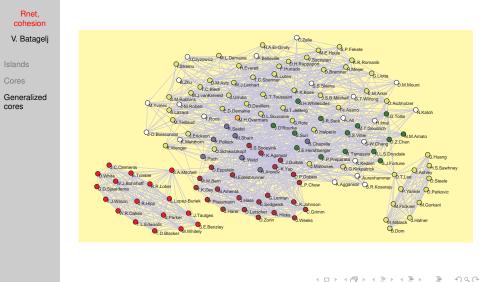
File/Network/Read [Geom.net] Network/Create Partition/k-Core/All Info/Partition Operations/Network+Partition/Extract Subnetwork [13-*] Draw/Network+First Partition Layout/Energy/Kamada-Kawai Options/Values of lines/Similarities Layout/Energy/Kamada-Kawai Operations/Network+Partition/Extract Subnetwork [21] Draw/Network Layout/Energy/Kamada-Kawai Options/Values of lines/Forget Layout/Energy/Kamada-Kawai [select Geom.net] Network/Create Vector/Generalized Cores/Sum/All Info/Vector Vector/Make Partition/by Intervals/Selected Thresholds [Info/Partition Operations/Network+Partition/Extract Subnetwork [2] Draw/Network Options/Values of lines/Similarities Layout/Energy/Fruchterman-Reingold

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Cores of orders 10–21 in Computational Geometry



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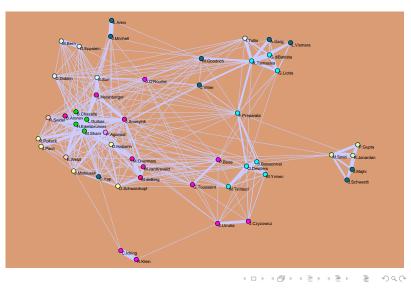
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p_S -core at level 46 of Geombib network



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