Temporal cores in networks

Vladimir Batagelj, Monika Cerinšek

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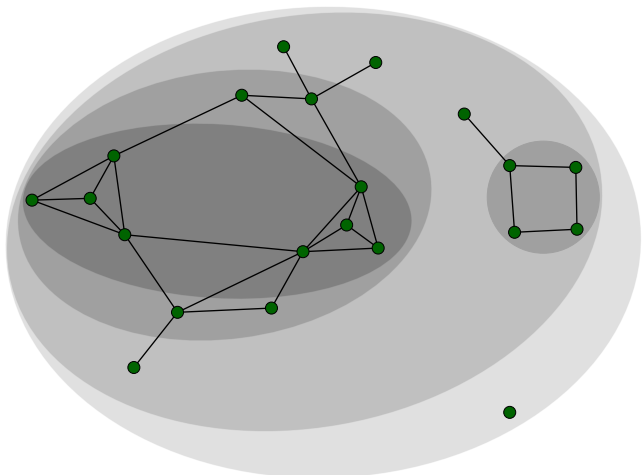
Core of order k

Network: $\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{P}, \mathcal{W})$; $n = |\mathcal{V}|, m = |\mathcal{L}|$

k-core (Seidman 1983): A subgraph $\mathcal{H} = (\mathcal{C}, \mathcal{L}(\mathcal{C}))$ induced by the set $\mathcal{C}$ is a k-core or a core of order k iff $\forall v \in \mathcal{C} : deg_{\mathcal{H}}(v) \geq k$ and $\mathcal{H}$ is the maximum subgraph with this property.

The core of maximum order – main core.
The core number of node v is the highest order of a core that contains this node.

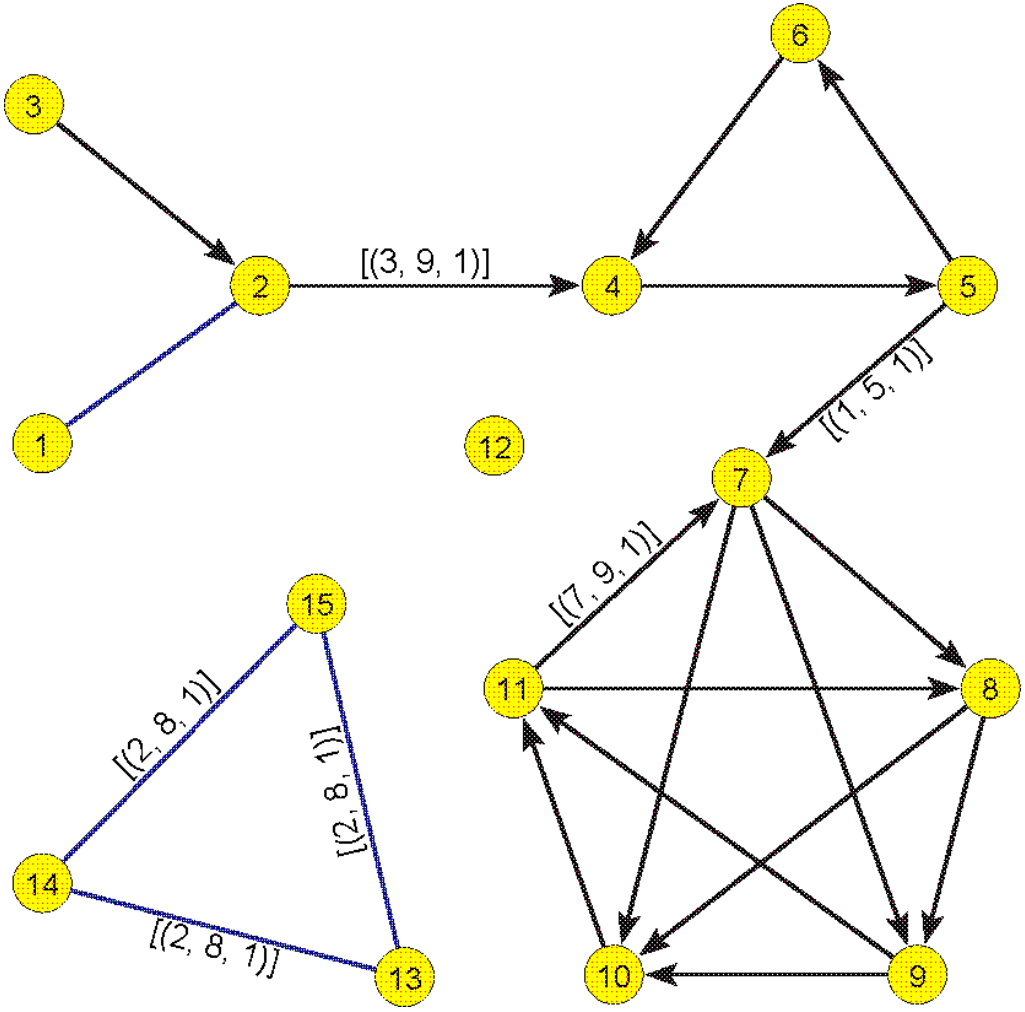


Core decomposition

```
1  CoreDecomposition(𝒩):
2  C = V
3  k = 1
4  while C ≠ ∅:
5          while ∃ u ∈ C ∋: deg(u) < k:
6                  for v ∈ N(u, C):
7                          deg(v) = deg(v) − 1
8                  C = C\v
9                  core(u) = k − 1
10         k = k + 1
```

Temporal network

Temporal network $\mathcal{N}_{\mathcal{T}} = (\mathcal{V}, \mathcal{L}, \mathcal{T}, \mathcal{P}, \mathcal{W})$ is obtained by attaching the time $\mathcal{T}$ to an ordinary network, where $\mathcal{T}$ is a set of time points: $t \in \mathcal{T}$ which are usually integers or reals.



Temporal quantities

Notion: $T(v)$ – the activity set of time points for the node v; $T(l)$ the activity set of time points for the link l

Consistency condition: If a link $l(u, v)$ is active at the time point t then its end-nodes u and v should be active at the time t :

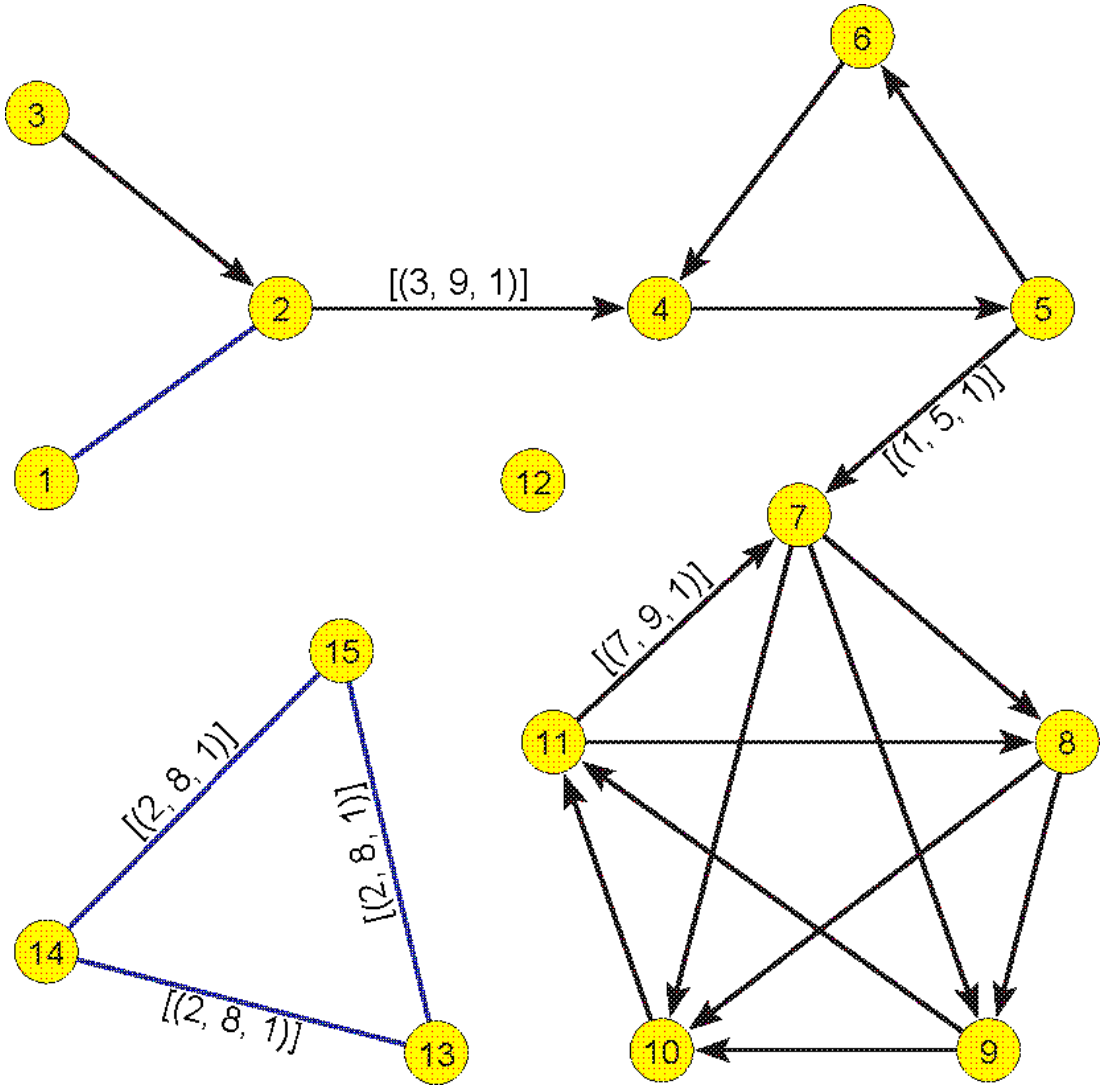
$$T(l(u, v)) \subseteq T(u) \cap T(v).$$

Temporal quantity a with the activity set $T_a \subseteq \mathcal{T}$ describes the changes of properties of nodes and links:

$$a = \begin{cases} \text{a'(t)} & \text{t} \in T_a \\ \text{undefined} & \text{t} \in \mathcal{T} \setminus T_a \end{cases}$$

Core maintenance

The problem of maintaining core numbers for a temporal network.

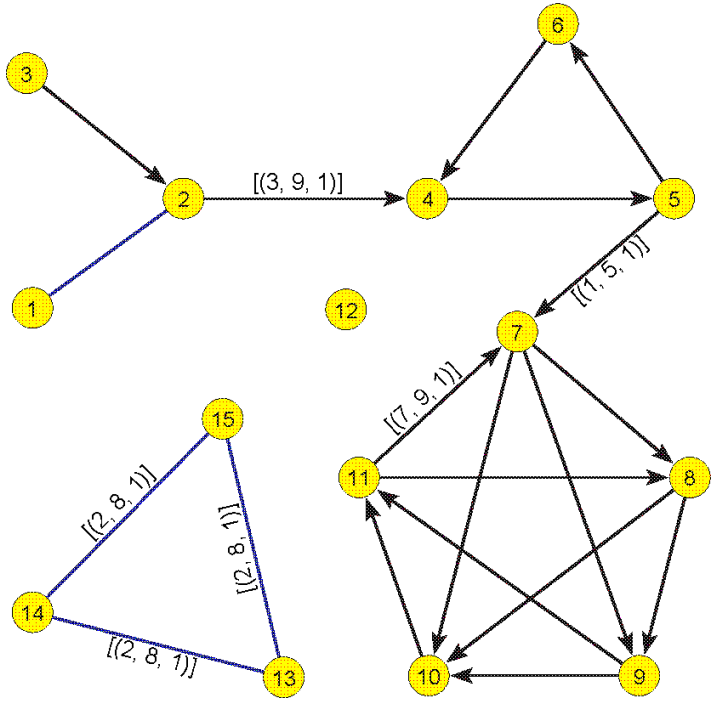


Simple algorithm for cores in temporal networks

```
1   TemporalCores(𝒩):
2   D = {u: [triples (start, finish, deg)]}
3   CoreHierarchy = {u: [triples with deg = 0]}
4   D = (D.filter(deg > 0)).remove(empty triples)
5   Dmin = {u: min deg}
6   while D not empty:
7           (dmin, u) = (deg, u) ∋: (u, deg) ∈ Dmin ∧ deg is min deg
8           core = [triples from D[u] ∋: deg[u] from triple is equal to dmin]
9           CoreHierarchy[u].add(core)
10          change = core.set(deg = −1)
11          D[u] = D[u].add(change).cutAt(dmin) \\ value >= dmin
12          for l in 𝒩.star(u):
13                  v = other end−node of l
14                  if not v in D: continue
15                  changeLink = l.intersection(change).set(deg = −1)
16                  if changeLink empty: continue
17                  diff = D[v].add(changeLink).cutAt(0) \\ value >= 0
18                  D[v] = diff.set(max(currentValue, dmin))
19                  if D[v] is empty:
20                          delete D[v], Dmin[v]
21                  else:
22                          Dmin[v] = triple ∈ D[v] with min deg
23          if D[u] empty:
24                  delete D[u], Dmin[u]
25          else:
26                  Dmin[u] = triple ∈ D[u] with min deg
27  return CoreHierarchy
```

Artificial example

Node	Degree	Core number
1	(1, 9, 1)	(1, 9, 1)
2	(1, 3, 2), (3, 9, 3)	(1, 9, 1)
3	(1, 9, 1)	(3, 9, 1)
4	(1, 3, 2), (3, 9, 3)	(1, 9, 2)
5	(1, 5, 3), (5, 9, 2)	(1, 9, 2)
6	(1, 9, 2)	(1, 9, 2)
7	(1, 5, 4), (5, 7, 3), (7, 9, 4)	(1, 7, 3), (7, 9, 4)
8	(1, 9, 4)	(1, 7, 3), (7, 9, 4)
9	(1, 9, 4)	(1, 7, 3), (7, 9, 4)
10	(1, 9, 4)	(1, 7, 3), (7, 9, 4)
11	(1, 7, 3), (7, 9, 4)	(1, 7, 3), (7, 9, 4)
12	(1, 9, 0)	(1, 9, 0)
13	(1, 2, 0), (2, 8, 2), (8, 9, 0)	(1, 2, 0), (2, 8, 2), (8, 9, 0)
14	(1, 2, 0), (2, 8, 2), (8, 9, 0)	(1, 2, 0), (2, 8, 2), (8, 9, 0)
15	(1, 2, 0), (2, 8, 2), (8, 9, 0)	(1, 2, 0), (2, 8, 2), (8, 9, 0)



Real-life example – Reuters terror news network[1]

Obtained from the CRA (Centering Resonance Analysis) networks produced by Steve Corman and Kevin Dooley at Arizona State University.

Based on all the stories released during 66 consecutive days by the news agency Reuters concerning the September 11 attack on the U.S., beginning at 9:00 AM EST 9/11/01.

Nodes: important words (terms), n = 13332

Lines: two nodes appear in the same utterance, m = 243447, undirected, weight is equal to the frequency of appearance, 50859 of them have the weight larger than 1. No loops.

Example: induced subnetwork on 50 most active nodes.

[1] Data available at: http://vlado.fmf.uni-lj.si/pub/networks/data/CRA/terror.htm

Real-life example – Reuters terror news network

Node Degree

1 (1, 2, 5), (2, 3, 6), (3, 4, 3), (4, 5, 5), (5, 6, 4), (6, 8, 3), (8, 10, 5), (10, 11, 3), (11, 13, 2), (13, 16, 3), (16, 17, 4), (17, 18, 5), (18, 19, 3), (19, 21, 1), (21, 22, 2), (22, 23, 1), (23, 24, 4), (24, 25, 1), (25, 29, 3), (29, 31, 2), (31, 33, 3), (33, 34, 1), (34, 36, 3), (36, 37, 2), (37, 39, 3), (39, 40, 4), (40, 41, 2), (41, 42, 0), (42, 43, 3), (43, 44, 2), (44, 45, 3), (45, 46, 1), (46, 47, 2), (47, 48, 3), (48, 49, 0), (49, 50, 4), (50, 51, 1), (51, 52, 2), (52, 53, 1), (53, 54, 0), (54, 58, 2), (58, 59, 3), (59, 60, 2), (60, 61, 4), (61, 62, 0), (62, 64, 2), (64, 65, 1), (65, 67, 2)

2 (1, 2, 27), (2, 3, 29), ..., (63, 64, 2), (64, 65, 0), (66, 67, 0)

...

50 (1, 2, 3), (2, 3, 2), (3, 5, 1), (5, 8, 0), (8, 10, 1), (10, 11, 2), (11, 12, 1), (12, 15, 0), (15, 16, 3), (16, 17, 1), (17, 19, 0), (19, 20, 1), (20, 21, 2), (21, 22, 0), (22, 24, 1), (24, 26, 0), (26, 27, 2), (27, 28, 0), (28, 29, 1), (29, 31, 0), (31, 32, 1), (32, 33, 0), (33, 35, 1), (35, 37, 0), (37, 38, 1), (38, 42, 0), (43, 44, 2), (44, 49, 0), (49, 50, 2), (51, 57, 0), (58, 61, 0), (61, 62, 1), (62, 67, 0)

Node Core number

1 (1, 2, 4), (2, 3, 5), (3, 5, 3), (5, 6, 4), (6, 8, 3), (8, 10, 4), (10, 11, 3), (11, 14, 2), (14, 18, 3), (18, 19, 2), (19, 21, 1), (21, 22, 2), (22, 23, 1), (23, 24, 3), (24, 25, 1), (25, 28, 2), (28, 29, 3), (29, 33, 2), (33, 34, 1), (34, 38, 2), (38, 39, 3), (39, 41, 2), (41, 42, 0), (42, 45, 2), (45, 46, 1), (46, 47, 2), (47, 48, 3), (48, 49, 0), (49, 50, 3), (50, 51, 1), (51, 52, 2), (52, 53, 1), (53, 54, 0), (54, 57, 2), (57, 58, 1), (58, 59, 2), (59, 60, 1), (60, 61, 2), (61, 62, 0), (62, 64, 2), (64, 65, 1), (65, 67, 2)

2 (1, 3, 5), (3, 6, 4), (6, 7, 5), ..., (63, 64, 1), (64, 65, 0), (66, 67, 0)

...

50 (1, 3, 2), (3, 5, 1), (5, 8, 0), (8, 10, 1), (10, 11, 2), (11, 12, 1), (12, 15, 0), (15, 16, 3), (16, 17, 1), (17, 19, 0), (19, 20, 1), (20, 21, 2), (21, 22, 0), (22, 24, 1), (24, 26, 0), (26, 27, 1), (27, 28, 0), (28, 29, 1), (29, 31, 0), (31, 32, 1), (32, 33, 0), (33, 35, 1), (35, 37, 0), (37, 38, 1), (38, 42, 0), (43, 44, 1), (44, 49, 0), (49, 50, 2), (51, 57, 0), (58, 61, 0), (61, 62, 1), (62, 67, 0)

Real-life example – Reuters terror news network

Node		Core number (≥ 3)
1	united_states	(1, 2, 4), (2, 3, 5), (5, 6, 4), (8, 10, 4)
2	attack	(1, 3, 5), (3, 6, 4), (6, 7, 5), (7, 10, 4), (11, 12, 4), (30, 31, 4)
4	people	(1, 3, 5), (3, 6, 4), (6, 7, 5), (7, 8, 4)
5	afghanistan	(1, 3, 4), (5, 6, 4), (6, 7, 5), (8, 10, 4), (30, 31, 4)
6	bin_laden	(1, 4, 4), (5, 6, 4), (6, 7, 5), (7, 10, 4), (11, 12, 4)
7	new_york	(1, 3, 5), (3, 6, 4), (6, 7, 5), (30, 31, 4)
8	pres_bush	(1, 3, 5), (3, 6, 4), (6, 7, 5), (7, 10, 4), (11, 12, 4)
9	washington	(1, 3, 5), (3, 6, 4), (6, 7, 5), (7, 10, 4), (11, 12, 4)
10	official	(1, 3, 5), (3, 4, 4), (5, 6, 4), (6, 7, 5)
12	military	(1, 2, 4), (5, 6, 4), (30, 31, 4)
13	plane	(1, 3, 5), (3, 7, 4)
14	world_trade_ctr	(1, 3, 5), (3, 6, 4), (6, 7, 5), (30, 31, 4)
15	security	(1, 2, 4), (2, 3, 5), (5, 6, 4)
16	american	(2, 3, 4)
17	country	(1, 3, 4), (5, 10, 4)
18	city	(1, 3, 5), (3, 4, 4)
19	war	(1, 2, 4), (2, 3, 5), (5, 8, 4)
20	tuesday	(1, 3, 5), (3, 7, 4)
21	pentagon	(1, 3, 5), (3, 4, 4), (5, 6, 4), (6, 7, 5)
22	force	(5, 6, 4)
23	government	(1, 3, 4), (5, 6, 4)
24	leader	(1, 4, 4), (6, 10, 4)

Node		Core number (≥ 3)
25	world	(1, 3, 5), (3, 10, 4)
26	terrorism	(2, 3, 4)
27	day	(2, 3, 4), (5, 6, 4)
28	week	(5, 6, 4), (6, 7, 5), (8, 10, 4), (11, 12, 4)
29	worker	(1, 2, 4), (2, 3, 5)
30	office	(1, 3, 4)
31	group	(2, 3, 4), (6, 7, 4)
32	air	(2, 3, 4), (5, 6, 4)
34	time	(1, 3, 5), (3, 4, 4), (5, 6, 4), (7, 8, 4)
35	hijack	(2, 3, 4)
36	strike	(2, 3, 4), (5, 6, 4), (6, 7, 5), (30, 31, 4)
38	flight	(2, 3, 4)
39	tell	(2, 3, 4)
40	terrorist	(1, 3, 4), (6, 7, 4)
41	airport	(2, 3, 4)
42	pakistan	(2, 3, 4), (5, 7, 4)
43	tower	(1, 3, 5), (3, 4, 4), (6, 7, 5)
45	new	(2, 3, 4)
47	wednesday	(2, 3, 5), (3, 4, 4), (8, 10, 4)
48	nation	(1, 3, 4), (5, 6, 4)
49	police	(2, 4, 4), (5, 6, 4)

Real-life example – Stem cell research[2]

A data set on the stem cell research during 1997–2012 in Spain collected by Gisela Cantos-Mateos consisting of data on papers about stem cell research in the SCI (Science Citation Index).

Nodes: Spanish institutions, n = 577

Lines: collaborations between institutions, m = 8578.

[2]Cantos-Mateos, G., Zulueta, M.A., Vargas-Quesada, B., Chinchilla-Rodrıguez, Z., 2014. Estudio evolutivo de la investigacion espanola con celulas madre. Visualizacion e identificacion de las principales l'ıneas de investigacion. El Profesional de la Informacion, 23(3), 259-271

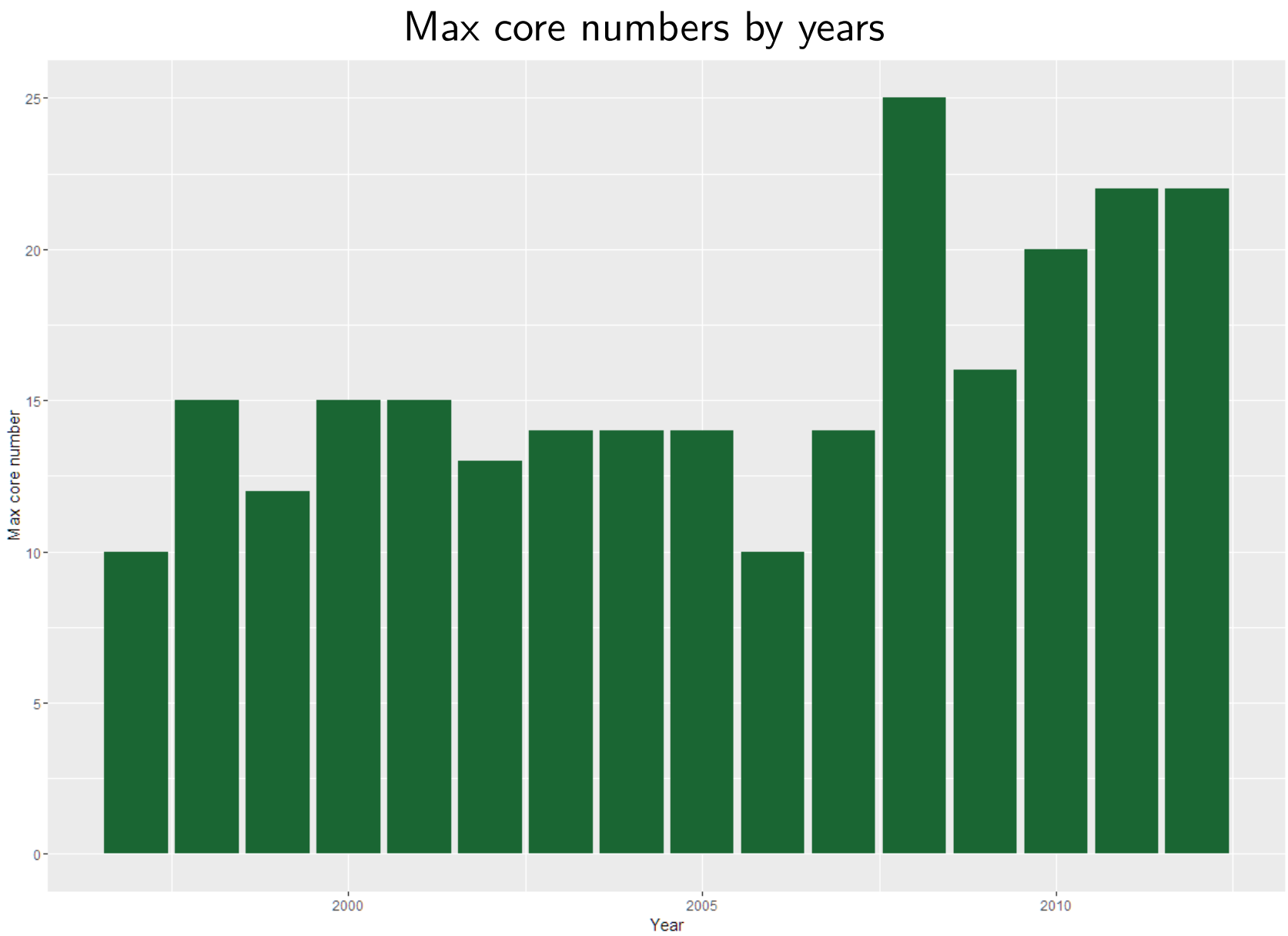
Real-life example – Stem cell research

Node		Core number ($\geq$ 20)
2	HCSC/M	(2010, 2011, 20), (2011, 2012, 21)
6	HUS/SA	(2008, 2009, 25), (2010, 2011, 20), (2011, 2012, 21), (2012, 2013, 22)
10	UNIZAR/Z	(2008, 2009, 21), (2012, 2013, 21)
13	HNJ/M	(2010, 2011, 20), (2012, 2013, 22)
259	HMS/Z	(2011, 2012, 21)
22	HDM/B	(2008, 2009, 21), (2012, 2013, 22)
27	HISC3/M	(2008, 2009, 25), (2010, 2011, 20), (2011, 2013, 22)
289	HJXXIII/T	(2008, 2009, 25)
36	UAB/B	(2008, 2009, 21), (2010, 2011, 20), (2011, 2013, 22)
40	HCL/B	(2010, 2011, 20), (2011, 2013, 22)
51	HBST/B	(2008, 2009, 25), (2011, 2012, 21)
55	HUPH/M	(2011, 2012, 21), (2012, 2013, 22)
266	URL/B	(2012, 2013, 22)
65	HRS/CO	(2012, 2013, 21)
69	UMA/MA	(2008, 2009, 21), (2010, 2011, 20), (2011, 2012, 21), (2012, 2013, 22)
74	CIBERDEM	(2008, 2009, 25)
77	UPV/V	(2008, 2009, 21)
82	UM/MU	(2008, 2009, 25)
344	HSO/M	(2011, 2012, 21)
92	GENYO/GR	(2011, 2012, 21)
272	ULEON/LE	(2011, 2013, 22)
103	CIBEROBN	(2011, 2012, 21)
111	HANDERSON/M	(2011, 2012, 21)
123	H-JAEN	(2012, 2013, 22)
133	HCSOL/MA	(2012, 2013, 22)
535	SERIDA/O	(2011, 2012, 21)
149	IIBM/M	(2011, 2012, 22)
176	HUVR/SE	(2008, 2009, 25), (2011, 2013, 22)
452	HVS/TO	(2011, 2012, 21)
232	HUB/B	(2008, 2009, 25)

Node		Core number ($\geq$ 20)
3	IN/A	(2008, 2009, 25)
8	IDIBELL/B	(2011, 2012, 20)
11	USAL/SA	(2008, 2009, 25), (2010, 2011, 20), (2011, 2013, 22)
16	ICO/CT	(2008, 2009, 25), (2010, 2011, 20), (2012, 2013, 22)
20	UPC/B	(2011, 2012, 21), (2012, 2013, 22)
23	UNAV	(2008, 2009, 25), (2011, 2013, 22)
543	PFIZER/M	(2011, 2012, 21)
34	HCL/V	(2010, 2011, 20), (2011, 2013, 22)
37	US/SE	(2010, 2011, 20)
46	IDIBAPS/B	(2008, 2009, 21), (2010, 2011, 20), (2011, 2013, 22)
53	H12O/M	(2008, 2009, 25), (2011, 2013, 21)
57	HCLB/Z	(2011, 2012, 21)
62	UAM/M	(2008, 2009, 25), (2010, 2011, 20), (2011, 2013, 22)
66	HCRUCES/BI	(2011, 2012, 21)
72	HUMV/S	(2008, 2009, 25), (2011, 2013, 22)
75	SEHH	(2011, 2012, 21), (2012, 2013, 20)
336	TERCEL	(2008, 2009, 25)
85	UA/A	(2008, 2009, 25), (2011, 2012, 20)
89	UPF/B	(2008, 2009, 21), (2012, 2013, 22)
93	CBMSO/M	(2010, 2011, 20), (2011, 2012, 22), (2012, 2013, 21)
310	SESCAM/TO	(2011, 2012, 21)
108	HGJF/CA	(2011, 2012, 21)
112	INCYL	(2008, 2009, 21), (2010, 2011, 20), (2012, 2013, 21)
124	HJC/C	(2011, 2012, 20)
134	IBV/V	(2008, 2009, 25)
146	HSC/GR	(2010, 2011, 20)
150	UNIOVI/O	(2010, 2011, 20)
186	UVA	(2012, 2013, 22)
80	HUPLFV/V	(2008, 2009, 25), (2010, 2011, 20), (2011, 2013, 22)
492	UPNA/NA	(2012, 2013, 22)

Node		Core number ($\geq$ 20)
5	CIC-IBMCC/SA	(2010, 2011, 20), (2011, 2013, 22)
9	UB/B	(2008, 2009, 25), (2010, 2011, 20), (2011, 2013, 22)
12	HVH/B	(2010, 2011, 20), (2011, 2013, 22)
17	HMM/MU	(2011, 2012, 22)
21	ICREA/B	(2010, 2011, 20)
24	UPV-EHU	(2008, 2009, 21), (2010, 2011, 20)
32	HRYC/M	(2008, 2009, 21), (2010, 2011, 20), (2011, 2013, 22)
35	HUGTIP/B	(2010, 2011, 20), (2012, 2013, 20)
38	UV/V	(2008, 2009, 25), (2010, 2011, 20), (2011, 2013, 22)
48	HSCSP/B	(2008, 2009, 21), (2010, 2011, 20), (2011, 2013, 22)
54	CNB	(2012, 2013, 22)
58	HCUN/NA	(2011, 2013, 22)
63	UCM/M	(2008, 2009, 25), (2010, 2011, 20), (2011, 2013, 22)
67	CIPF/V	(2008, 2009, 21)
73	UGR/GR	(2011, 2012, 22), (2012, 2013, 20)
76	HULP/M	(2008, 2009, 25), (2010, 2011, 20), (2011, 2013, 22)
81	HVA/MU	(2011, 2012, 20), (2012, 2013, 21)
87	HUP/M	(2011, 2013, 22)
91	CIBERNED	(2012, 2013, 22)
96	BACM/GR	(2011, 2013, 22)
102	USC	(2011, 2013, 22)
109	HVN/GR	(2008, 2009, 21), (2011, 2012, 22), (2012, 2013, 21)
258	INIA/M	(2012, 2013, 22)
403	SERGAS/C	(2008, 2009, 25)
135	CRG/B	(2008, 2009, 25), (2011, 2012, 21)
147	HGM/M	(2010, 2011, 20), (2011, 2013, 22)
153	UAH/M	(2008, 2009, 25)
192	IRB/B	(2011, 2012, 22)
307	HVB/LE	(2010, 2011, 20)
253	UCLM	(2011, 2012, 21), (2012, 2013, 22)

Real-life example – Stem cell research



Real-life example – Violence network[3]

Roberto Franzosi collected from the journal news in the period January 1919 – December 1922 information about the different types of interactions between political parties and other groups of people in Italy. The violence network contains only the data about violent actions and counts the number of interactions per month.

Nodes: groups of people, n = 29

Links: violent interactions, m = 105

[3] Franzosi, R., 1997. Mobilization and CounterMobilization Processes: From the Red Years (1919-20) to the Black Years (1921-22) in Italy. A New Methodological Approach to the Study of Narrative Data. Theory and Society, 26(2-3), 275-304

Real-life example – Violence network

Node		**Core number** (≥ 3)
16	workers	(29, 30, 3), (33, 34, 3), (39, 41, 3)
1	undefined	(29, 30, 3), (39, 40, 3)
2	?	(31, 32, 3), (33, 34, 3), (40, 41, 3)
3	people	(31, 32, 3), (33, 34, 3), (39, 40, 3)
4	police	(31, 32, 3), (33, 34, 3), (40, 41, 3)
21	catholics	(33, 34, 3)
7	fascists	(29, 30, 3), (31, 32, 3), (33, 34, 3), (39, 41, 3)
8	communists	(29, 30, 3)
10	socialists	(31, 32, 3), (40, 41, 3)

Real-life example – Violence network

Node		**Core number** (≥ 2)
1	undefined	(15, 16, 2), (17, 18, 2), (25, 29, 2), (29, 30, 3), (31, 32, 2), (38, 39, 2), (39, 40, 3), (41, 44, 2), (45, 46, 2), (48, 49, 2)
2	?	(14, 16, 2), (17, 18, 2), (28, 29, 2), (31, 32, 3), (32, 33, 2), (33, 34, 3), (34, 35, 2), (40, 41, 3)
3	people	(16, 18, 2), (23, 24, 2), (25, 26, 2), (28, 30, 2), (31, 32, 3), (33, 34, 3), (35, 37, 2), (39, 40, 3), (41, 43, 2), (48, 49, 2)
4	police	(11, 12, 2), (14, 20, 2), (21, 23, 2), (29, 31, 2), (31, 32, 3), (32, 33, 2), (33, 34, 3), (34, 37, 2), (38, 40, 2), (40, 41, 3)
5	land owners	(15, 16, 2), (17, 20, 2), (29, 30, 2), (36, 37, 2), (38, 40, 2), (42, 43, 2)
7	fascists	(11, 12, 2), (16, 17, 2), (19, 20, 2), (21, 24, 2), (25, 29, 2), (29, 30, 3), (30, 31, 2), (31, 32, 3), (32, 33, 2), (33, 34, 3), (34, 37, 2), (38, 39, 2), (39, 41, 3), (41, 44, 2), (45, 46, 2), (48, 49, 2)
8	communists	(28, 29, 2), (29, 30, 3), (31, 33, 2), (35, 37, 2), (43, 44, 2)
9	workers (agr)	(15, 16, 2), (17, 20, 2), (28, 30, 2), (31, 32, 2), (33, 35, 2), (38, 43, 2), (45, 46, 2)
10	socialists	(11, 12, 2), (16, 18, 2), (19, 20, 2), (22, 23, 2), (25, 26, 2), (27, 30, 2), (31, 32, 3), (33, 37, 2), (38, 40, 2), (40, 41, 3), (41, 42, 2)
12	war affected	(35, 36, 2), (39, 40, 2)
13	protesters	(15, 16, 2), (21, 22, 2), (29, 30, 2), (31, 32, 2), (38, 40, 2)
16	workers	(11, 12, 2), (14, 18, 2), (19, 20, 2), (21, 24, 2), (25, 26, 2), (27, 29, 2), (29, 30, 3), (30, 33, 2), (33, 34, 3), (34, 37, 2), (38, 39, 2), (39, 41, 3), (41, 44, 2), (45, 46, 2)
17	the right	(17, 18, 2), (41, 42, 2)
19	populars	(41, 42, 2)
20	students	(17, 18, 2)
21	catholics	(33, 34, 3)
25	republicans	(26, 27, 2)
26	thugs	(29, 30, 2)
27	prisoners/arrested	(40, 41, 2)

Conclusions

Improve the complexity of the algorithm

Extend the algorithm to generalized temporal cores

Find user friendly presentations of results

Compare with the streaming core algorithms

Thank you!

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dr. Vladimir Batagelj, Institute of Mathematics, Physics and Mechanics, Ljubljana, Slovenia & University of Primorska, Koper, Slovenia
vladimir.batagelj@fmf.uni-lj.si

dr. Monika Cerinšek, Abelium d.o.o R&D, Ljubljana, Slovenia
monika@abelium.eu